
FRM PART I BOOK 1: FOUNDATIONS OF RISK MANAGEMENT

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FRM PART I BOOK 1: FOUNDATIONS OF RISK MANAGEMENT

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INTRODUCTION TO THE 2012 KAPLAN SCHWESER STUDY NOTES FRM EXAM PART I

Thank you for trusting Kaplan Schweser to help you reach your career and education goals. We are very pleased to be able to help you prepare for the 2012 FRM Exam. In this introduction, I want to explain what is included in the Study Notes, suggest how you can best use Kaplan Schweser materials to prepare for the exam, and direct you toward other educational resources you will find helpful as you study for the exam.

Study Notes—A 4-book set that includes complete coverage of all risk-related topic areas and AIM statements, as well as Concept Checkers (multiple-choice questions for every assigned reading) and Challenge Problems (exam-like questions). In addition, the Study Notes include background material for a number of key FRM-related concepts (these background readings supplement the curriculum). At the end of each book, we have included relevant questions from past GARP FRM practice exams. These old exam questions are a great tool for understanding the format and difficulty of actual exam questions.

To help you master the FRM material and be well prepared for the exam, we offer several additional educational resources, including:

8-Week Online Class—Live online program (eight 3-hour sessions) that is offered each week, beginning in March for the May exam and September for the November exam. The online class brings the personal attention of a classroom into your home or office with 24 hours of real-time instruction led by either Dr. John Paul Broussard, CFA, FRM, PRM or Dr. Greg Filbeck, CFA, FRM, CAIA. The class offers in-depth coverage of difficult concepts, instant feedback during lecture and Q&A sessions, and discussion of past FRM exam questions. Archived classes are available for viewing at any time throughout the study season. Candidates enrolled in the Online Class also have access to downloadable slide files and **Instructor E-mail Access**, where they can send questions to the instructor at any time.

If you have purchased the Schweser Study Notes as part of the Essential, Premium, or PremiumPlus Solution, you will also receive access to **Instructor-led Office Hours**. Office Hours allow you to get your FRM-related questions answered in real time and view questions from other candidates (and faculty answers) as well. Office Hours is a text-based, live, interactive, online chat with the weekly online class instructor. Archives of previous Instructor-led Office Hours sessions are sorted by topic and are posted shortly after each session.

Practice Exams—The Practice Exam Book contains two full-length, 100-question (4-hour) exams. These exams are important tools for gaining the speed and confidence you will need to pass the exam. Each exam contains answer explanations for self-grading. Also, by entering your answers at Schweser.com, you can use our Performance Tracker to find out how you have performed compared to other Kaplan Schweser FRM candidates.

Interactive Study Calendar—Use your Online Access to tell us when you will start and what days of the week you can study. The Interactive Study Calendar will create a study plan just for you, breaking each topic area into daily and weekly tasks to keep you on track and help you monitor your progress through the FRM curriculum.

Online Question Database—In order to retain what you learn, it is important that you quiz yourself often. We offer download and online versions of our FRM SchweserPro Qbank, which contains over 1,000 practice questions and explanations for Part I of the FRM Program.

In addition to these study products, there are many educational resources available at Schweser.com, including the **FRM Video Library** and the **FRM Exam-tips Blog**. Just log into your account using the individual username and password that you received when you purchased the Schweser Study Notes.

How to Succeed

The FRM exam is a formidable challenge, and you must devote considerable time and effort to be properly prepared. You must learn the material, know the terminology and techniques, understand the concepts, and be able to answer at least 70% of the questions quickly and correctly. 250 hours is a good estimate of the study time required on average, but some candidates will need more or less time depending on their individual backgrounds and experience. To provide you with an overview of the FRM Part I curriculum, we have included a list of all GARP assigned readings in the order they appear in our Study Notes. Every topic in our Notes is cross-referenced to an FRM assigned reading, so should you require additional clarification with certain concepts, you can consult the appropriate assigned reading.

There are no shortcuts to studying for this exam. Expect GARP to test you in a way that will reveal how well you know the FRM curriculum. You should begin studying early and stick to your study plan. You should first read the Study Notes and complete the Concept Checkers for each topic. At the end of each book, you should answer the provided Challenge Problems and practice exam questions to understand how concepts have been tested in the past. You can also attend our 8-Week Online Class to assist with retention of the exam concepts. You should finish the overall curriculum at least two weeks before the FRM exam. This will allow sufficient time for Practice Exams and further review of those topics that you have not yet mastered.

Best wishes for your studies and your continued success,

Eric Smith

Eric Smith, CFA, FRM
Senior Project Manager
Kaplan Schweser

GARP 2012 FRM PART I STUDY GUIDE

FOUNDATIONS OF RISK MANAGEMENT

Part I Exam Weight: 20%

Philippe Jorion, *Value-at-Risk: The New Benchmark for Managing Financial Risk, 3rd Edition* (New York: McGraw-Hill, 2007).

1: Chapter 1 – The Need for Risk Management

Edwin J. Elton, Martin J. Gruber, Stephen J. Brown and William N. Goetzmann, *Modern Portfolio Theory and Investment Analysis, 8th Edition* (Hoboken, NJ: John Wiley & Sons, 2009).

2: Chapter 5 – Delineating Efficient Portfolios

3: Chapter 13 – The Standard Capital Asset Pricing Model

4: Chapter 14 – Nonstandard Forms of Capital Asset Pricing Models

5: Chapter 16 – The Arbitrage Pricing Model APT – A New Approach to Explaining Asset Prices

Noel Amenc and Veronique Le Sourd, *Portfolio Theory and Performance Analysis* (West Sussex, England: John Wiley & Sons, 2003).

6: Chapter 4 – Applying the CAPM to Performance Measurement: Single-Index Performance Measurement Indicators

7: Casualty Actuarial Society, Enterprise Risk Management Committee, “Overview of Enterprise Risk Management,” May 2003.

René M. Stulz, *Risk Management & Derivatives* (Florence, KY: Thomson South-Western, 2002).

8: Chapter 3 – Creating Value with Risk Management

Steve Allen, *Financial Risk Management: A Practitioner's Guide to Managing Market and Credit Risk* (New York: John Wiley & Sons, 2003).

9: Chapter 4 – Financial Disasters

10: René M. Stulz, “*Risk Management Failures: What Are They and When Do They Happen?*” Fisher College of Business Working Paper Series (Oct. 2008).

11: GARP Code of Conduct (available on GARP website)

QUANTITATIVE ANALYSIS

Part I Exam Weight: 20%

James Stock and Mark Watson, *Introduction to Econometrics*, Brief Edition (Boston: Pearson Education, 2008).

12: Chapter 2 – Review of Probability

13: Chapter 3 – Review of Statistics

14: Chapter 4 – Linear Regression with One Regressor

15: Chapter 5 – Regression with a Single Regressor: Hypothesis Tests and Confidence Intervals

16: Chapter 6 – Linear Regression with Multiple Regressors

17: Chapter 7 – Hypothesis Tests and Confidence Intervals in Multiple Regression

Svetlozar Rachev, Christian Menn, and Frank Fabozzi, *Fat-Tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection and Option Pricing* (Hoboken, NJ: John Wiley & Sons, 2005).

18: Chapter 2 – Discrete Probability Distributions

19: Chapter 3 – Continuous Probability Distributions

Philippe Jorion, *Value-at-Risk: The New Benchmark for Managing Financial Risk*, 3rd Edition.

20: Chapter 12 – Monte Carlo Methods

John Hull, *Options, Futures, and Other Derivatives*, 8th Edition (New York: Pearson Prentice Hall, 2012).

21: Chapter 22 – Estimating Volatilities and Correlations

Linda Allen, Jacob Boudoukh, Anthony Saunders, *Understanding Market, Credit and Operational Risk: The Value at Risk Approach* (Oxford: Blackwell Publishing, 2004).

22: Chapter 2 – Quantifying Volatility in VaR Models

FINANCIAL MARKETS AND PRODUCTS

Part I Exam Weight: 30%

John Hull, *Options, Futures, and Other Derivatives, 8th Edition*.

- 23: Chapter 1 – Introduction
- 24: Chapter 2 – Mechanics of Futures Markets
- 25: Chapter 3 – Hedging Strategies using Futures
- 26: Chapter 4 – Interest Rates
- 27: Chapter 5 – Determination of Forward and Futures Prices
- 28: Chapter 6 – Interest Rate Futures
- 29: Chapter 7 – Swaps
- 30: Chapter 10 – Properties of Stock Options
- 31: Chapter 11 – Trading Strategies Involving Options

Helyette Geman, *Commodities and Commodity Derivatives: Modeling and Pricing for Agriculturals, Metals and Energy* (West Sussex, England: John Wiley & Sons, 2005).

- 32: Chapter 1 – Fundamentals of Commodity Spot and Futures Markets: Instruments, Exchanges and Strategies

Robert L. McDonald, *Derivatives Markets, 2nd Edition* (Boston: Addison-Wesley, 2006).

- 33: Chapter 6 – Commodity Forwards and Futures

Anthony Saunders and Marcia Millon Cornett, *Financial Institutions Management: A Risk Management Approach, 7th Edition* (New York: McGraw-Hill, 2010).

- 34: Chapter 14 – Foreign Exchange Risk

Frank Fabozzi, *The Handbook of Fixed Income Securities, 7th Edition* (New York: McGraw-Hill, 2005).

- 35: Chapter 13 – Corporate Bonds

VALUATION AND RISK MODELS

Part I Exam Weight: 30%

Bruce Tuckman, *Fixed Income Securities, 2nd Edition* (Hoboken, NJ: John Wiley & Sons, 2002).

36: Chapter 1 – Bond Prices, Discount Factors, and Arbitrage

37: Chapter 2 – Bond Prices, Spot Rates, and Forward Rates

38: Chapter 3 – Yield to Maturity

39: Chapter 5 – One-Factor Measures of Price Sensitivity

John Hull, *Options, Futures, and Other Derivatives, 8th Edition*.

40: Chapter 12 – Binomial Trees

41: Chapter 14 – The Black-Scholes-Merton Model

42: Chapter 18 – The Greek Letters

Kevin Dowd, *Measuring Market Risk, 2nd Edition* (West Sussex, England: John Wiley & Sons, 2005).

43: Chapter 2 – Measures of Financial Risk

Linda Allen, Jacob Boudoukh, Anthony Saunders, *Understanding Market, Credit and Operational Risk: The Value at Risk Approach*.

44: Chapter 3 – Putting VaR to Work

John Hull, *Risk Management and Financial Institutions, 2nd Edition* (Boston: Pearson Prentice Hall, 2010).

45: Chapter 18 – Operational Risk

Linda Allen, Jacob Boudoukh, Anthony Saunders, *Understanding Market, Credit and Operational Risk: The Value at Risk Approach*.

46: Chapter 5 – Extending the VaR Approach to Operational Risk

Philippe Jorion, *Value-at-Risk: The New Benchmark for Managing Financial Risk, 3rd Edition*.

47: Chapter 14 – Stress Testing

48: “Principles for Sound Stress Testing Practices and Supervision” (Basel Committee on Banking Supervision Publication, Jan 2009).

John Caouette, Edward Altman, Paul Narayanan and Robert Nimmo, *Managing Credit Risk: The Great Challenge for the Global Financial Markets, 2nd Edition* (New York: John Wiley & Sons, 2008).

49: Chapter 6 – The Rating Agencies

Arnaud de Servigny and Olivier Renault, *Measuring and Managing Credit Risk*, (New York: McGraw-Hill, 2004).

50: Chapter 2 – External and Internal Ratings, including the Appendix

J. Caouette, E. Altman, P. Narayanan, R. Nimmo, *Managing Credit Risk, 2nd Edition*.

51: Chapter 23 – Country Risk Models

Michael Ong, *Internal Credit Risk Models: Capital Allocation and Performance Measurement*, (London: Risk Books, 2003).

52: Chapter 4 – Loan Portfolios and Expected Loss

53: Chapter 5 – Unexpected Loss

BACKGROUND READINGS

In addition to the assigned material, we have included background topics that will assist you in understanding the assigned concepts. For more information on these background topics, see the following readings:

Time Value of Money – *Quantitative Methods for Investment Analysis, 2nd Edition*, Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle, “The Time Value of Money,” Chapter 1.

VaR Methods – Philippe Jorion, *Value-at-Risk: The New Benchmark for Managing Financial Risk, 3rd Edition*, Chapter 10.

Interest Rate Derivative Instruments – *Derivatives and Portfolio Management*, CFA Program Curriculum, Volume 6, Level 2 (CFA Institute, 2010).

THE NEED FOR RISK MANAGEMENT

Topic 1

EXAM FOCUS

In this topic, we present an overview of core risk management concepts which will be discussed throughout the FRM curriculum. A majority of these concepts will be discussed in much more detail in later topics in the Schweser Study Notes. This material examines the types of risk faced by financial institutions and important tools that can be used to manage these risks.

Corporations need to apply risk management techniques in order to combat increases in financial risk. Financial risks must be managed carefully because they have the potential to cause large losses. The derivatives market has played a key role in assisting businesses with both hedging risks and speculating on risks. However, derivatives must be used with caution, as the potential for large catastrophic losses exists in the absence of proper regulation.

MAJOR SOURCES OF RISK

AIM 1.1: Define risk and describe some of the major sources of risk.

AIM 1.2: Differentiate between business and financial risks and give examples of each.

Risk is defined as the unexpected variability of asset prices and/or earnings. There are two major sources of risk: business and financial.

Business risk is the risk that a firm is subjected to during daily operations and includes the risks that result from business decisions and the business environment. Business risk includes strategic risk, which reflects the risks inherent in the decisions of senior management in setting a business strategy. Also included in business risk are the macroeconomic risks that impact a firm's operations and sales. The ability to effectively manage business risk is a core competency for stronger firms. An example of a business risk is the risk that the economy will slow and demand for a product will fall.

Financial risks are the result of a firm's financial market activities. An example of financial risk is interest rate movements after the issuance of floating-rate bonds. In this case, the issuing firm will be negatively impacted if market rates increase. Another example of financial risk is suffering a loss from the default of a financial obligation.

EXTREME MARKET MOVEMENTS

AIM 1.3: Relate significant market events of the past several decades to the growth of the risk management industry.

Several recent and significant historical events have increased the volatility of financial markets, thereby raising the need for financial risk management systems. Examples of extreme market events include:

- 1971: Fixed exchange rate system broke down.
- 1973: Shocks to price of oil, high inflation, and volatile interest rates.
- 1987: Black Monday, which saw a 23% decline in U.S. stock prices.
- 1989: Japanese stock market bubble deflated.
- 1997: Asian contagion decimated Asian equity markets.
- 1998: Russian debt default and the collapse of the Long-Term Capital Management hedge fund.
- 2001: The September 11 attacks on the World Trade Center and Pentagon set in motion the 2001 U.S. equity market collapse.
- 2007–2009: Credit crisis resulting from mortgage market meltdown and huge amounts of bank leverage.

It is evident that these events caused significant increases to volatility which resulted in huge financial losses. Appropriate use of financial risk management tools serve to provide protection against potential future losses.

In addition to increases in volatility, firms have recently become more exposed to economic and financial variables. Two major factors have led to increases in the sensitivity to these financial factors: deregulation and globalization. Before the 1970s, banks were heavily regulated, and regulations such as interest rate ceilings reduced bank exposure to interest rate fluctuations. **Deregulation** in banks, therefore, led to increases in interest rate sensitivity. **Globalization** led to firms doing business outside their respective domestic borders causing these firms to have more exposure to currency changes and international competition. These changes have increased the importance of risk management because financial institutions are now exposed to a wider variety of risks.

Risk arises from many different sources. For example, it can be human-created (inflation or war), unforeseen (earthquakes or hurricanes), or result from economic growth spurred on by technological innovations. Regarding growth through innovation, a process known as creative destruction replaces old goods with new ones that are more efficient and effective. It promotes economic growth by forcing companies to continue to produce better products and services. Economic growth depends on taking risks so, therefore, risk should not be viewed as something we must avoid, but as something we must manage carefully.

FUNCTIONS AND PURPOSES OF FINANCIAL INSTITUTIONS

AIM 1.4: Describe the functions and purposes of financial institutions as they relate to financial risk management.

Financial institutions serve as financial intermediaries for managing financial risk. Financial institutions create markets and instruments to share and hedge risks, provide risk advisory services, and act as a counterparty by assuming the risk of others. Because of the roles that financial institutions serve, the institutions must excel at measuring and pricing financial risk.

Financial institutions and markets, unfortunately, cannot protect against all risk as some risks remain difficult to hedge. Take, for instance, the risk that arises from government interference in credit markets or foreign exchange markets. Improper allocation of credit or the fixing of exchange rates inappropriately can lead to adverse economic conditions. Fixing exchange rates will reduce currency fluctuations; however, governments must also balance the effects of this exchange rate mechanism on monetary and fiscal policy and international trade and investment.

DERIVATIVES

AIM 1.5: Define what a derivative contract is and how it differs from a security.

Derivatives can be used in financial risk management to efficiently hedge and/or manage financial risks. A **derivative contract** is a contract that derives its value from an underlying security. Derivatives have a finite, predefined life, a predefined reference rate or price, and a predefined notional amount. Securities (i.e., stocks and bonds) are issued to raise capital in order to support projects that will earn a return greater than the cost of those securities. Derivatives on the other hand are not issued to raise capital and are considered zero-sum games. This means that in a derivative contract, the losses from one side of the transaction will equal the other side's gains.

Leverage allows derivatives to be useful as hedging instruments due to their low transaction costs and limited initial cash outlay. The downside to leverage is the “double-edged sword” nature of this financial tool. As leverage increases, the variability of returns increases.

The derivatives market continues to evolve in response to the growing risks facing corporations. Financial engineering has led to the development of new derivative contracts such as credit default swaps and stock index futures, which have helped to address emerging risks and opportunities.

FINANCIAL RISK MANAGEMENT

AIM 1.6: Define financial risk management.

Financial risk management is the process of detecting, assessing, and managing financial risks. There are many types of risks that will be defined later in this topic (e.g., market risk) as well as several tools for managing these risks. One of the major tools used to manage market, credit, and operational risk is the **value at risk (VaR)** measure.

VALUE AT RISK

AIM 1.7: Define value at risk (VaR) and describe how it is used in risk management.

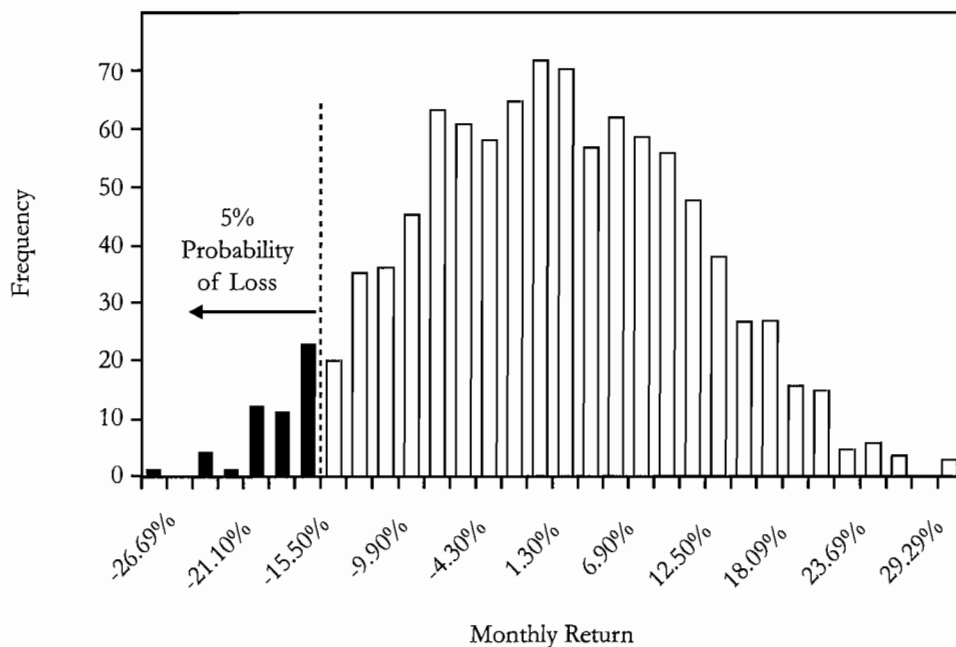
Value at risk (VaR) is defined as the maximum loss over a defined period of time at a stated level of confidence, given normal market conditions. VaR corresponds to the loss in the tail of the return distribution. VaR is a key statistical measure utilized by many financial institutions.

To illustrate this measure, assume you have gathered 1,000 monthly returns for a security and produced the histogram shown in Figure 1. You decide that you want to compute the monthly VaR for this security at a confidence level of 95%. At a 95% confidence level, the lower tail displays the lowest 5% of the underlying distribution's returns. For this distribution, the value associated with a 95% confidence level is a return of -15.5%. If you have \$1,000,000 invested in this security, the one month VaR is \$155,000 ($-15.5\% \times \$1,000,000$).



Professor's Note: This is an example of historical VaR. In Book 2, we will discuss three types of VaR: delta-normal VaR, historical VaR, and Monte Carlo VaR.

Figure 1: Histogram of Monthly Returns



Other Risk Management Tools

AIM 1.8: Describe the advantages and disadvantages of VaR relative to other risk management tools such as stop-loss limits, notional limits, and exposure limits.

A **stop-loss limit** seeks to limit the amount of loss on a position by eliminating the position after a cumulative loss threshold has been exceeded. It is a control mechanism that functions ex-post (i.e., after the loss has occurred). This measure is easy to calculate, easy to explain, and can be aggregated across assets (i.e., it allows for risk to be measured across an entire portfolio/institution).

A **notional limit** is a limit on the notional amount invested in a position or asset. This measure fails to explain the risk of a position to changes in risk factors. For example, two bonds with the same notional amount will likely have two different risk levels. Notional limits are easy to calculate and explain, but cannot be aggregated across assets.

Exposure limits are limits to risk factor exposures. For interest rates, the applicable exposure is duration. For equity market exposure, the relevant exposure is beta. For options, a major exposure is delta. While these measures identify the exposure of an asset to an applicable risk factor, the measures fail to quantify the volatility of the risk factors and the correlations between risk factors. Exposure limits are difficult to calculate, difficult to explain, and cannot be combined across assets.

VaR is an ex-ante (i.e., before the fact) measure and can at times be difficult to calculate. However, it does capture exposures to risk factors and accounts for variation and covariation in risk factors. VaR is comparable across different business units in a firm with different assets and risk characteristics. That is, VaR is interpreted the same, regardless of the assets in question. VaR is also frequently used in the risk budgeting process, where upper management allocates a risk level to each asset class.

Although VaR is easily understood and usually widely accepted, all methods for calculating VaR first require accurate inputs, and this issue becomes more and more daunting as the number of assets in a portfolio gets larger. Just identifying all risks (without actually predicting their impacts on portfolio value) may be infeasible.

Valuation and Risk Management Using VaR

AIM 1.9: Compare and contrast valuation and risk management, using VaR as an example.

Valuation is the process of discounting the expected future value of an asset to determine the current price of the asset. The expected value for an asset is the mean value for the distribution of possible values. The valuation of derivatives requires risk-neutral pricing so that arbitrage situations will not persist.

VaR as a risk management tool attempts to explain the possible future distribution of asset values with specific focus on the lower tail of the return distribution. VaR looks at the future

value of an asset, not the present value, and utilizes the distribution of returns that is often assumed to be equivalent to the historical distribution. Less precision is required in VaR analysis than in valuation because as long as the model is not biased, errors will tend to offset each other.

TYPES OF RISK

AIM 1.10: Define and describe the four major types of financial risks: market, liquidity, credit, and operational; and their forms.

Market risk is the risk that declining prices or volatility of prices in the financial markets will result in a loss. There are two major types of market risk: absolute risk and relative risk.

Liquidity risk is the possibility of sustaining significant losses due to the inability to sufficiently liquidate a position at a fair price.

Credit risk is the possibility of default by a counterparty in a financial transaction, and the monetary exposure to credit risk is a function of the probability of default and the loss that results given default occurs.

Operational risk is the risk of loss due to inadequate monitoring systems, management failure, defective controls, fraud, and/or human errors. Operational risk is particularly relevant to derivatives trading, because derivatives are inherently highly leveraged instruments, which enable traders to expose a firm to enormous losses using a relatively small amount of capital.

Market Risk

Absolute risk focuses on the volatility of total returns. **Relative risk** is referred to as tracking error since it is usually measured relative to a benchmark index or portfolio.

Directional risks are linear risk exposures in economic or financial variables (e.g., interest rates, stock indices). **Non-directional risks** are risks that have non-linear exposures or neutral exposures to changes in economic or financial variables.

Basis risk is the risk that the price of a hedging instrument and the price of the asset being hedged are not perfectly correlated. An example of basis risk is using a put option to hedge an equity exposure. In this case, the option position will have to be monitored and adjusted appropriately since the change in the put option will likely not be exactly equal to the change in the equity price.

The risk of loss from changes in actual or implied volatility of market prices is known as **volatility risk**. The volatility of equity indices or interest rates may change due to market events, significant investor uncertainty, political instability, or structural changes in the economy. Firms with exposure to equity markets may see significant losses if there is an unexpected change in volatility.

Liquidity Risk

Liquidity risk includes both funding liquidity risk and asset-liquidity risk. **Asset-liquidity risk**, which is sometimes called market (or trading) liquidity risk, results from a large position size forcing transactions to influence the price of securities. To manage asset-liquidity risk, limits can be established on assets that are not heavily traded.

Funding liquidity risk, which is sometimes called cash-flow risk, refers to the risk that a financial institution will be unable to raise the cash necessary to roll over its debt; to fulfill the cash, margin, or collateral requirements of counterparties; or to meet capital withdrawals.

Credit Risk

For credit risk, exposure is the size or value of loss that would be realized if a credit event occurred. The recovery rate is the percentage of assets that could be recovered from a counterparty after a credit event occurs.

A **credit event** relates to a change in a counterparty's ability to perform its previously agreed to financial obligations. Market prices incorporate changes to credit ratings or changes to default probabilities, which can be looked at as both market risk and credit risk. Therefore, instances can exist where a change in price is due to market and credit risk.

Sovereign risk refers to the risks resulting from a country's actions. Sovereign risk differs from the other forms of credit risk in that it is country specific. A country's willingness and ability to repay its obligations are often factors looked at when evaluating the sovereign risk of foreign government debt. The sources of sovereign risk stem from a country's political and legal systems.

Settlement is the exchange of two payments or the exchange of an asset for payment. **Settlement risk** is the risk that a counterparty will fail to deliver its obligation after the party has made its delivery. Presettlement risk is lower than settlement risk because, with this measure, payments will offset (i.e., are netted). On the other hand, settlement risk exposure deals with the full value of each payment.

Operational Risk

Operational, market, and credit risk are interrelated. An operational failure may increase market and credit risks. A bank that engages in buying and selling derivatives without an adequate understanding of the derivatives market could suffer significant losses. Those losses could then result in a change in credit rating for the firm and a reduction in market price for its securities.

Model risk is the risk of loss due to the use of misspecified or misapplied models. An institution buying or selling collateralized mortgage obligations (CMOs) may be exposed to model risk if the model used to price the CMOs does not adequately account for the probability of default in the underlying mortgages.

People risk relates to the risk associated with fraud perpetrated by internal employees and/or external individuals. An example of people risk is a rogue trader within an institution that intentionally falsifies reports related to losses incurred.

Legal risk is the risk of a loss in value due to legal issues including lawsuits, fines, penalties, and/or damages. An example of legal risk is when a counterparty sues a bank to avoid meeting its obligations. Legal risks are managed through appropriate corporate policies developed by legal counsel in conjunction with a firm's financial risk managers. Legal risks are inherent in doing business but can be controlled through corporate policies and procedures. Ineffective policies or procedures open a firm up to substantial legal risk.

KEY CONCEPTS

1. Risk is the unexpected variability of asset prices or earnings. The two major categories of risk are business and financial.
2. Business risks are the risks that a firm assumes through its daily operations and financial risks are the result of a firm's financial market activities.
3. Significant historical events have increased the volatility of financial markets and caused significant financial losses.
4. Financial institutions serve as financial intermediaries for financial risk by creating markets and instruments to share and hedge risks, providing risk advisory services, and acting as a counterparty by assuming the risk of others.
5. A derivatives contract is a contract that derives its value from an underlying security, has a finite, defined life, a defined reference rate, and is defined for a specific notional amount.
6. The benefit of leverage in derivatives is that leverage makes derivatives useful for hedging and speculation because of the low transaction costs and limited upfront investment required. The downside is that the small initial investment makes it difficult to assess downside risk.
7. Value at risk (VaR) is defined as the maximum loss over a defined period of time at a stated level of confidence.
8. A stop-loss limit seeks to limit the amount of loss on a position by eliminating the position after a cumulative loss threshold has been exceeded. A notional limit is a limit on the notional amount invested in a position or asset. Exposure limits are limits to risk factor exposures. VaR gives the maximum loss over a defined period of time at a stated level of confidence, given normal market conditions.
9. Market risk is the risk that declining prices or volatility of prices in the financial market will result in a loss. Liquidity risk is the possibility of sustaining significant losses due to the inability to take or liquidate a position at a fair price. Credit risk is the possibility of default by the counterparty to a financial transaction. Operational risk is the risk of loss due to inadequate monitoring systems, management failure, defective controls, fraud, or human errors.
10. Absolute risk is the volatility of total returns, while relative risk is measured relative to a benchmark index or portfolio.
11. Basis risk is the risk that the price of a hedging instrument and the price of the asset being hedged are not perfectly correlated.
12. Volatility risk is the risk of loss from changes in actual or implied volatility of market prices.
13. Asset-liquidity risk results from a large position size in an asset relative to the asset's typical trading lot size causing a transaction to heavily influence market prices. Funding liquidity risk refers to the risk that a financial institution will be unable to raise the cash necessary to roll over its debt; to fulfill the cash, margin, and collateral requirements of counterparties; and to meet capital withdrawals.

14. Credit exposure is the size of loss that would be realized if a credit event occurred. The recovery rate is the percentage of assets that could be recovered from the counterparty after the credit event occurs.
15. A credit event relates to a change in a counterparty's ability to perform its previously agreed to financial obligations. Market prices incorporate changes to credit ratings or changes to default probabilities, which can be looked at as both market risk and credit risk.
16. Sovereign risk is country specific risk that results from a country's actions. A country's willingness and ability to repay its obligations are often factors looked at when evaluating the sovereign risk of foreign government debt.
17. Settlement risk is the risk that a counterparty will fail to deliver its obligation after delivery of one side has been made.
18. Model risk is the risk of loss due to the use of misspecified or misapplied financial models.
19. People risk relates to the risk associated with fraud perpetrated by individuals internal (i.e., employees) and/or external to the institution.
20. Legal risk is the risk of a loss stemming from legal issues such as lawsuits, fines, penalties, and/or damages. Legal risks are managed through policies and procedures developed by legal counsel and risk managers.

CONCEPT CHECKERS

1. Which of the following scenarios is an example of business risk?
 - A. An error by a derivatives trader causes a significant loss.
 - B. A significant market downturn causes a firm's pension plan to experience significant losses.
 - C. A global recession has led to a decrease in demand for a business's products.
 - D. Interest rates increase causing a company to have to make higher coupon payments on its floating-rate debt.

2. Which of the following statements is most likely correct regarding the function(s) of financial institutions in risk management? Financial institutions:
 - I. create markets and instruments to hedge financial risks.
 - II. act as a counterparty by assuming the risk of others.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.

3. Which of the following is least likely to have been a contributing factor for the increase in financial risk management awareness?
 - A. Deregulation.
 - B. Globalization.
 - C. Nationalization.
 - D. The shift from a fixed to a floating-rate exchange system.

4. Which of the following statements is correct regarding valuation and value at risk (VaR)?
 - A. Valuation and VaR are both concerned with the mean of a return distribution.
 - B. Valuation and VaR are both focused on the tails of the return distribution.
 - C. Valuation looks at the tails of a return distribution, while VaR looks at the mean.
 - D. Valuation looks at the mean of the return distribution, while VaR looks at the lower tail.

5. The risk that the price of a hedging instrument and the price of the asset being hedged are not perfectly correlated is referred to as:
 - A. basis risk.
 - B. volatility risk.
 - C. correlation risk.
 - D. directional risk.

CONCEPT CHECKER ANSWERS

1. C A macroeconomic change that affects the core business operations is a business risk. An error by a derivatives trader is an example of an operational financial risk. A downturn in the market causing losses to a pension plan is an example of a market related risk. Interest rate increases that lead to a firm making higher coupon payments is another example of market risk.
2. C Financial institutions perform both of these functions related to financial risk management.
3. C Globalization and deregulation increased firm exposure to market related volatility, which contributed to an increase in the importance of financial risk management. The move from a fixed to floating-rate currency exchange system created volatility in exchange rates leading to a greater need for exchange rate risk management. Nationalization was not a contributing factor to the increased importance of financial risk management.
4. D Valuation is focused on the mean of the returns distribution, while VaR concentrates on those returns in the lower tail.
5. A Basis risk is the risk that the price of a hedging instrument and the price of the asset being hedged are not perfectly correlated.

DELINEATING EFFICIENT PORTFOLIOS

Topic 2

EXAM FOCUS

This topic addresses fundamental concepts regarding portfolio return and volatility. You should be able to calculate expected return and volatility for a two-asset portfolio and understand the importance of correlation in portfolio diversification. It is important to understand the shape of the portfolio possibilities curve and what is meant by the minimum variance portfolio. Additionally, you will need to know what the efficient frontier is and how short sales and riskless borrowing affect it.

EXPECTED RETURN AND VOLATILITY OF A TWO-ASSET PORTFOLIO

AIM 2.1: Calculate the expected return and volatility of a portfolio of risky assets.

The expected return on a portfolio is a weighted average of the expected returns on the individual assets that are included in the portfolio. For example, for a two-asset portfolio:

$$E(R_p) = w_1E(R_1) + w_2E(R_2)$$

where:

$E(R_p)$ = expected return on Portfolio P

w_i = proportion (weight) of the portfolio allocated to Asset i

$E(R_i)$ = expected return on Asset i

The weights (w_1 and w_2) must sum to 100% for a two-asset portfolio.

The variance of a two-asset portfolio equals:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}_{1,2}$$

where:

σ_p^2 = variance of the returns for Portfolio P

σ_1^2 = variance of the returns for Asset 1

σ_2^2 = variance of the returns for Asset 2

w_i = proportion (weight) of the portfolio allocated to Asset i

$\text{Cov}_{1,2}$ = covariance between the returns of the two assets

The covariance, $\text{Cov}_{1,2}$, measures the strength of the relationship between the returns earned on assets 1 and 2. The covariance is unbounded (ranges from negative infinity to positive infinity); therefore, it is not a very useful measure of the strength of the relationship between two asset's returns. Instead, we often scale the covariance by the standard deviations of the two assets to derive the correlation coefficient, $\rho_{1,2}$:

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2}$$

From the previous equation, notice that the covariance equals $\rho_{1,2} \sigma_1 \sigma_2$. Therefore, the variance of the two-asset portfolio can also be written as:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2$$

The portfolio standard deviation or portfolio volatility is the positive square root of the portfolio variance.

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2 \right]^{1/2}$$

Example: Expected return and volatility for a two-asset portfolio

Using the information in the following figure, calculate the expected return and standard deviation of the two-asset portfolio.

Characteristics for a Two-Stock Portfolio

	<i>Caffeine Plus</i>	<i>Sparklin'</i>
Amount invested	\$40,000	\$60,000
Expected return	11%	25%
Standard deviation	15%	20%
Correlation	0.30	

Answer:

First, determine the weight of each stock relative to the entire portfolio. Since the investments are \$40,000 and \$60,000, we know the total value of the portfolio is \$100,000:

$$w_c = \text{investment}/\text{portfolio value} = \$40,000 / \$100,000 = 0.40$$

$$w_s = \text{investment}/\text{portfolio value} = \$60,000 / \$100,000 = 0.60$$

Next, we determine the expected return on the portfolio:

$$E(R_p) = w_c E(R_c) + w_s E(R_s)$$

$$E(R_p) = (0.40)(0.11) + (0.60)(0.25) = 0.1940 = 19.40\%$$

Then, we calculate the variance of the portfolio:

$$\begin{aligned}\sigma_p^2 &= w_c^2 \sigma_c^2 + w_s^2 \sigma_s^2 + 2w_c w_s \rho_{cs} \sigma_c \sigma_s \\ &= (0.40)^2 (0.15)^2 + (0.60)^2 (0.20)^2 + 2(0.40)(0.60)(0.30)(0.15)(0.20) \\ &= 0.02232\end{aligned}$$

And, finally, the standard deviation of the portfolio:

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.02232} = 0.1494 = 14.94\%$$

THE PORTFOLIO POSSIBILITIES CURVE

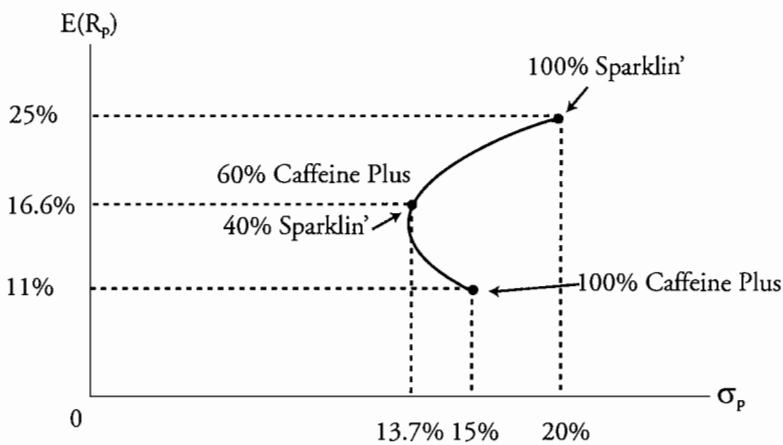
In the Caffeine Plus and Sparklin' example, we calculated the expected return and volatility of one possible combination: 40% in Caffeine Plus and 60% in Sparklin'. However, an infinite number of combinations of the two stocks are possible. We can plot these combinations on a graph with expected return on the y -axis and standard deviation on the x -axis, commonly referred to as plotting in risk/return "space." The graph of the possible portfolio combinations is referred to as the **portfolio possibilities curve**. Figure 1 shows some of these combinations.

Figure 1: Portfolio Returns for Various Weights of Two Assets

$w_{\text{Caffeine Plus}}$	100%	80%	60%	40%	20%	0%
$w_{\text{Sparklin'}}$	0%	20%	40%	60%	80%	100%
\hat{R}_p	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%
σ_p	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%

The plot in Figure 2 represents all possible expected return and standard deviation combinations attainable by investing in varying amounts of Caffeine Plus and Sparklin'.

Figure 2: Expected Return and Standard Deviation Combinations



There are several things to notice about Figure 2:

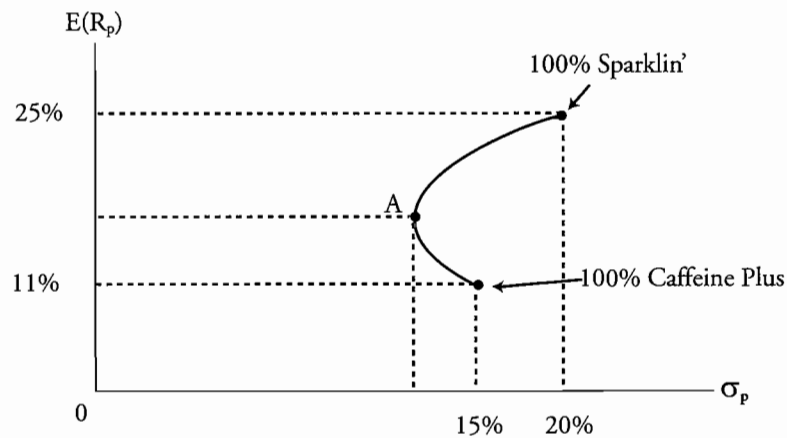
- If 100% of the portfolio is allocated to Caffeine Plus, the portfolio will have the expected return and standard deviation of Caffeine Plus (i.e., Caffeine Plus is the portfolio), and the investment return and risk combination is at the lower end of the curve.
- As the investment in Caffeine Plus is decreased and the investment in Sparklin' is increased, the investment moves up the curve to the point where the portfolio's expected return is 16.6% with a standard deviation of 13.72% (labeled 60% Caffeine Plus/40% Sparklin').
- Finally, if 100% of the portfolio is allocated to Sparklin', the portfolio will have the expected return and standard deviation of Sparklin', and the investment return and risk combination is at the upper end of the curve (e.g., higher risk and higher expected return).

MINIMUM VARIANCE PORTFOLIO

AIM 2.4: Define the minimum variance portfolio.

The **minimum variance portfolio** is the portfolio with the smallest variance among all possible portfolios on a portfolio possibilities curve. The minimum variance portfolio consisting of Caffeine Plus and Sparklin' contains approximately 70% Caffeine Plus and 30% Sparklin' and has an expected return of 15.3% and a standard deviation of 13.6%. On the portfolio possibilities curve, the minimum variance portfolio represents the left-most point on the curve. Figure 3 illustrates the minimum variance portfolio for Caffeine Plus and Sparklin' (point A).

Figure 3: Minimum Variance Portfolio



Professor's Note: We do not anticipate that you will be required to solve for the minimum variance portfolio mathematically, but it is included here for your reference.

Start with the expression for portfolio standard deviation, substituting $(1 - w_1)$ for w_2 :

$$\sigma_p = \left[w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2w_1(1 - w_1)\rho_{1,2}\sigma_1\sigma_2 \right]^{1/2}$$



Next, take the partial derivative of portfolio standard deviation with respect to w_1 and set the derivative equal to zero to solve for the weights of the minimum variance portfolio.

$$\frac{\partial \sigma_p}{\partial w_1} = \frac{2w_1\sigma_1^2 + 2w_1\sigma_2^2 - 2\sigma_2^2 - 4w_1\rho_{1,2}\sigma_1\sigma_2 + 2\rho_{1,2}\sigma_1\sigma_2}{2\left[w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2 + 2w_1(1 - w_1)\rho_{1,2}\sigma_1\sigma_2\right]^{1/2}} = 0$$

$$w_1 = \frac{[\sigma_2^2 - \rho_{1,2}\sigma_1\sigma_2]}{[\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2]}$$

$$w_2 = 1 - w_1$$

CORRELATION AND PORTFOLIO DIVERSIFICATION

AIM 2.2: Explain how covariance and correlation affect the expected return and volatility of a portfolio of risky assets.

Perfect Positive Correlation

In the case where two assets have perfect positive correlation (i.e., $\rho = 1$), the portfolio standard deviation reduces to the simple weighted average of the individual standard deviations indicating no diversification. This is shown mathematically as:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \times 1 \times \sigma_1 \sigma_2 \right]^{1/2} = w_1 \sigma_1 + w_2 \sigma_2$$

Since expected portfolio return is a linear combination of the individual asset returns, and risk is a linear combination of the individual asset volatilities, the portfolio possibilities curve for two perfectly correlated assets is a straight line. This line is given as:

$$E(R_p) = \left(E(R_2) - \frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2} \sigma_2 \right) + \left(\frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2} \right) \sigma_p$$

Professor's Note: Recognize that the portfolio possibilities curve for perfectly positively correlated assets is a straight line. For those interested in the algebra, the expression can be solved as follows:

Recognizing that the weights of the two assets must add to one; the weight of asset one in the portfolio standard deviation equation can be solved as follows:

$$\sigma_p = w_1 \sigma_1 + (1 - w_1) \sigma_2 \Rightarrow w_1 = \frac{\sigma_p - \sigma_2}{\sigma_1 - \sigma_2}$$



The portfolio possibilities curve can then be found by substituting the weight of asset one into the expected return equation as follows:

$$E(R_p) = \frac{\sigma_p - \sigma_2}{\sigma_1 - \sigma_2} E(R_1) + \left(1 - \frac{\sigma_p - \sigma_2}{\sigma_1 - \sigma_2} \right) E(R_2)$$

$$E(R_p) = \left(E(R_2) - \frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2} \sigma_2 \right) + \left(\frac{E(R_1) - E(R_2)}{\sigma_1 - \sigma_2} \right) \sigma_p$$

No diversification is achieved if the correlation between assets equals +1. As the correlation between two assets *decreases*, however, the benefits of diversification *increase*. As the correlation decreases, there is less tendency for stock returns to move together. The separate movements of each stock serve to reduce the volatility of a portfolio to a level that is less than the weighted sum of its individual components (e.g., less than $w_1 \sigma_1 + w_2 \sigma_2$).

Perfect Negative Correlation

The greatest diversification is achieved in the case where two assets have perfect negative correlation (i.e., $\rho = -1$). In this case, the portfolio standard deviation reduces to two linear equations, which are:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \times -1 \times \sigma_1 \sigma_2 \right]^{1/2} = w_1 \sigma_1 - w_2 \sigma_2 \text{ or } -w_1 \sigma_1 + w_2 \sigma_2$$

When two assets have perfect negative correlation, it is possible to construct a portfolio with zero volatility by setting the standard deviation equal to zero and solving for the portfolio weights. The portfolio with zero volatility has portfolio weights of:

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

$$w_2 = 1 - w_1$$

Given that the standard deviation reduces to two linear equations, the portfolio possibilities curve for two assets with perfect negative correlation will be two line segments.

Zero Correlation

When the correlation between two assets is zero, the covariance term in the portfolio standard deviation expression is eliminated, and the resulting expression is:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \times 0 \times \sigma_1 \sigma_2 \right]^{1/2} = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 \right]^{1/2}$$

In this case, the standard deviation expression reduces to a non-linear equation, and the portfolio possibilities curve will be non-linear.

Assuming that the standard deviations of the individual assets are greater than zero, it is impossible to construct a portfolio with zero volatility. The weights of the minimum variance portfolio can be solved as previously discussed. The weights are calculated as:

$$w_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$w_2 = 1 - w_1$$

Moderate Positive Correlation

Most equities are positively correlated (i.e., $0 < \rho < 1$). If we assume that two assets are moderately correlated (e.g., $\rho = 0.5$), then the portfolio standard deviation reduces to:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \times \frac{1}{2} \times \sigma_1 \sigma_2 \right]^{1/2} = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_1 w_2 \sigma_1 \sigma_2 \right]^{1/2}$$

Similar to the case of zero correlation, assets with moderate correlation have non-linear portfolio possibilities curves. To determine the minimum variance portfolio in this case, you would apply the formula discussed in the previous Professor's Note.

An Example of Correlation and Portfolio Diversification

To illustrate the effects of correlation on diversification, consider the expected return and standard deviation data derived for domestic stocks, DS, and domestic bonds, DB as shown in Figure 4.

Figure 4: Diversification Example

	<i>Expected Return</i>	<i>Standard Deviation</i>
Domestic Stocks (DS)	0.20	0.30
Domestic Bonds (DB)	0.10	0.15

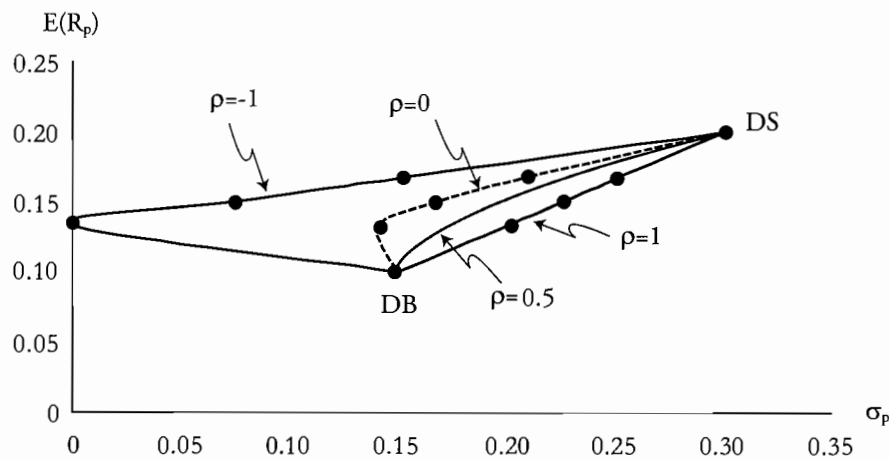
Figure 5 shows the expected return and standard deviation combinations for various portfolio percentage allocations to domestic stocks and domestic bonds for each of the correlations +1, 0.5, 0, and -1.

Figure 5: Expected Return/Standard Deviation Combinations for Various Allocations

<i>DS % Allocation</i>	<i>DB % Allocation</i>	$E(R_p)$	σ_p			
			$\rho = 1$	$\rho = 0.5$	$\rho = 0$	$\rho = -1$
100.00	0.00	0.200	0.300	0.300	0.300	0.300
66.67	33.33	0.167	0.250	0.229	0.206	0.150
50.00	50.00	0.150	0.225	0.198	0.168	0.075
33.33	66.67	0.133	0.200	0.173	0.141	0.000
0.00	100.00	0.100	0.150	0.150	0.150	0.150

Figure 6 shows the plot of the expected returns and standard deviations for each of the four correlations.

Figure 6: Effects of Correlation on Portfolio Risk



As indicated in Figure 6, the lower the correlation between the returns of the stocks in the portfolio, the greater the diversification benefits. If the correlation equals +1 (the solid black line), the minimum-variance frontier is a straight line between the two points (DB and DS), and there is no benefit to diversification. If the correlation equals -1 (the solid blue line), the minimum-variance frontier is two straight-line segments, and there exists a portfolio combination of stocks and bonds with a standard deviation of *zero* (the allocation of 66.67% to domestic bonds and 33.33% to domestic stocks).

THE SHAPE OF THE PORTFOLIO POSSIBILITIES CURVE

AIM 2.3: Describe the shape of the portfolio possibilities curve.

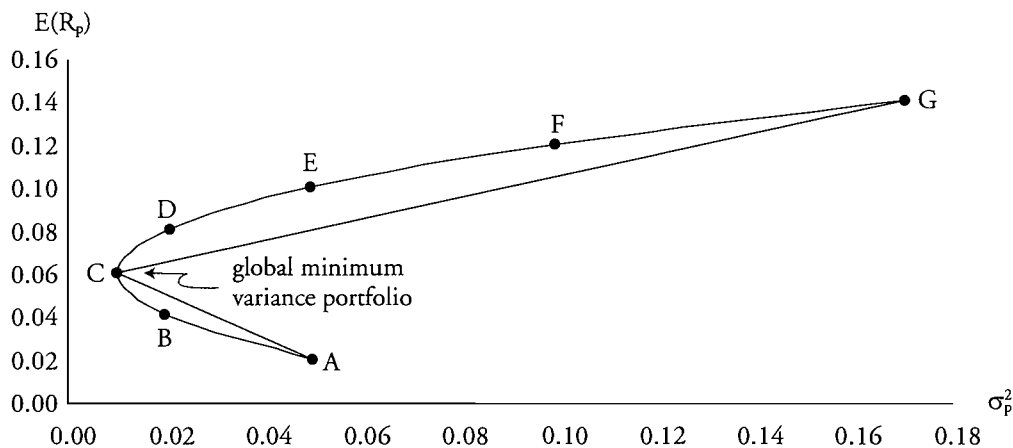


Professor's Note: For this AIM, we are not considering the special cases where the portfolio possibilities curve is a straight line (i.e., $\rho = 1$) or two line segments (i.e., $\rho = -1$). In all other cases, the portfolio possibilities curve is a curve similar to Figure 7.

Looking at Figure 7, the shape of the portfolio possibilities curve is best described in two pieces.

- The piece of the portfolio possibilities curve that lies above the minimum variance portfolio (from point C through point G) is concave.
- The piece of the portfolio possibilities curve that lies below the minimum variance portfolio (from point A through point C) is convex.

Figure 7: Shape of the Portfolio Possibilities Curve



Professor's Note: A concave function is one where the function lies above a straight-line segment connecting any two points on the function. A convex function lies below a straight-line segment connecting any two points on the function.



In Figure 7, the function is above the line segment from C to G. Therefore, the portion of the portfolio possibilities curve from C to G is concave. The function is below the line segment from A to C. Therefore, the portion of the portfolio possibilities curve from A to C is convex.

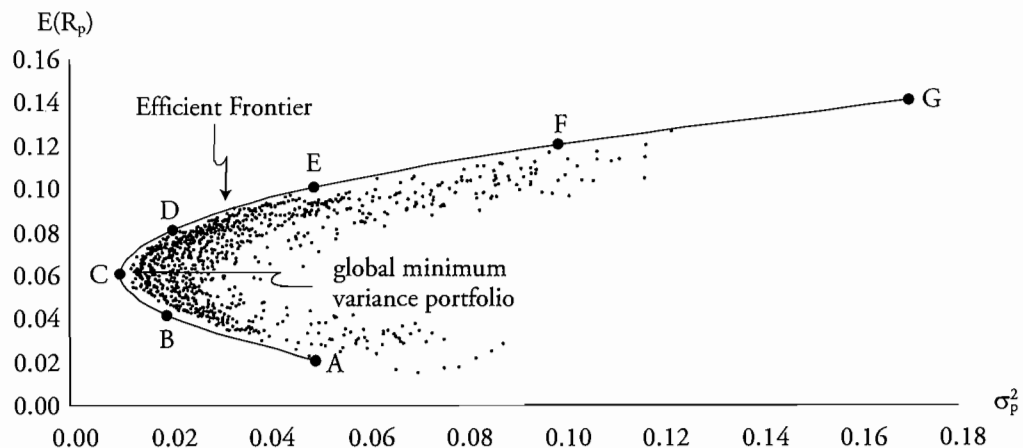
Another important aspect regarding the shape of the portfolio possibilities curve is that the curve must lie to the left of a line segment connecting any two points on the curve. From the discussion of portfolio diversification and correlation, combinations of two assets with perfect positive correlation result in a straight line. Combinations of assets with lower correlation will always lie to the left of that line.

THE EFFICIENT FRONTIER

AIM 2.5: Define the efficient frontier and describe the impact on it of various assumptions concerning short sales and borrowing.

Plotting all risky assets and potential combinations of risky assets will result in a graph similar to Figure 8.

Figure 8: Efficient Frontier



Notice that the graph includes some portfolios that no rational investor would select. All portfolios lying on the inside of the curve are inefficient. Additionally, some portfolios offer higher returns with identical risk. For example, portfolios A and E have identical risk; however, Portfolio E has a much higher expected return, and a similar contrast exists for Portfolio D versus Portfolio B. All rational investors would prefer Portfolio D over Portfolio B, and Portfolio E over Portfolio A.

Portfolios such as D and E are called **efficient portfolios**, which are portfolios that have:

- Minimum risk of all portfolios with the same expected return.
- Maximum expected return for all portfolios with the same risk.

The **efficient frontier** is a plot of the expected return and risk combinations of all efficient portfolios, all of which lie along the upper-left portion of the possible portfolios (from Point C to Point G in Figure 8).

Short Sales and the Efficient Frontier

When short sales are allowed, the shape of the efficient frontier changes. To examine how it changes, consider again the Caffeine Plus and Sparklin' example.

Referring back to the example, Caffeine Plus has an expected return of 11% and a standard deviation of 15%, and Sparklin' has an expected return of 25% and a standard deviation of 20%. The correlation between Caffeine Plus and Sparklin' is 0.30. Although neither stock has a negative return, it may make sense to short sell one of the stocks. In this case, Sparklin' has a higher expected return, so shorting Caffeine Plus and investing in Sparklin' would expand the efficient frontier. Figure 9 highlights the portfolio return and volatility for combinations of Sparklin' and Caffeine Plus including short sales.

Figure 9: Portfolio Returns for Various Weights of Two Assets (w/ Short Sales)

$w_{\text{Caffeine Plus}}$	100%	80%	60%	40%	20%	0%	-20%	-40%	-60%	-80%	-100%
$w_{\text{Sparklin'}}$	0%	20%	40%	60%	80%	100%	120%	140%	160%	180%	200%
\hat{R}_P	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%	27.80%	30.60%	33.40%	36.20%	39.00%
σ_P	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%	23.28%	26.82%	30.53%	34.36%	38.28%

When allowing for short sales, the efficient frontier expands up and to the right. By shorting, it is possible to create higher return and higher volatility portfolio combinations that would not be possible otherwise. Theoretically, with no limitations on shorting, it would be possible to construct a portfolio with infinite return.



Professor's Note: Up to this point, we have discussed risky assets. Now, we add the risk-free asset to the set of asset choices and examine the effect it has on investment choices.

Combining the Risk-Free Rate with the Efficient Frontier

So far, our portfolios have consisted of risky assets only. However, in reality, investors usually allocate their wealth across both risky and risk-free assets. The following discussion illustrates the effects of the inclusion of the risk-free asset. A risk-free asset is a security that has a return known ahead of time, so the variance of the return is zero.

Consider the task of creating portfolios comprising the risk-free asset, F , and a risky portfolio, P . Assume that Portfolio P lies on the efficient frontier of risky assets. Various combinations (weightings) of Portfolio P and the risk-free asset can be created. By adding the risk-free asset to the investment mix, a very important property emerges: *The shape of the efficient frontier changes from a curve to a line.*

Recall that the expected return for a portfolio of two assets equals the weighted average of the asset expected returns. Therefore, the expected return on Investment C that combines the risk-free asset and risky Portfolio P equals:

$$E(R_C) = w_F R_F + w_P E(R_P)$$

where:

w_F = percentage allocated to the risk-free asset

w_P = percentage allocated to Portfolio P

Also, recall that the variance of the portfolio of two assets (F and P) equals:

$$\sigma_C^2 = w_F^2 \sigma_F^2 + w_P^2 \sigma_P^2 + 2w_F w_P \text{Cov}_{FP}$$

where:

σ_C^2 = variance for Investment C

σ_F^2 = variance for the risk-free asset

σ_P^2 = variance for Portfolio P

Cov_{FP} = covariance between F and P

Observe that since we know that the variance and the standard deviation of the risk-free asset both equal zero, and that the covariance of the risk-free asset with any risky asset also equals zero, the equations for the variance and standard deviation for Investment C simplify to:

$$\sigma_C^2 = w_P^2 \sigma_P^2$$

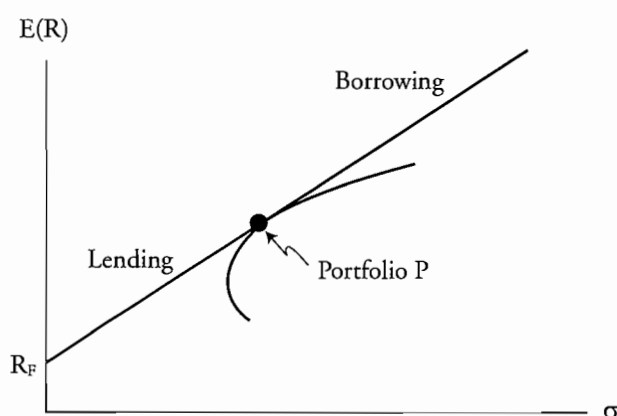
$$\sigma_C = w_P \sigma_P$$

Because the expected return and portfolio standard deviation of the combination of a risk-free asset and risky portfolio are both linear, the efficient frontier reduces to a linear equation. That is, by including the risk-free asset, we have caused the efficient frontier to become a straight line. The equation for the efficient frontier becomes the **capital market line (CML)**.

$$E(R_C) = R_F + \left(\frac{E(R_P) - R_F}{\sigma_P} \right) \sigma_C$$

Figure 10 illustrates the combination of the risk-free asset with the risky portfolio.

Figure 10: Efficient Frontier including the Risk-Free Asset



When the risk-free asset is combined with the risky Portfolio P, the efficient frontier becomes a line with:

- The intercept equal to the risk-free rate, and
- The slope equal to the reward-to-risk ratio for the risky portfolio.

Note that the capital market line is tangent to the efficient frontier. The point of tangency, Portfolio P, is known as the market portfolio. This portfolio contains all available risky assets in proportion to their total market values.

If all investors agree on the efficient frontier (i.e., they have homogeneous expectations regarding the risks and returns for all risky assets), they will hold a combination of the market portfolio and the risk-free asset. Risk-averse investors will create lower risk portfolios by lending (i.e., investing in the risk-free asset). More risk-tolerant investors will increase portfolio return by borrowing at the risk-free rate. This result is known as the separation theorem.

KEY CONCEPTS

1. The expected return for a two-asset portfolio is:

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

The portfolio standard deviation or portfolio volatility for a two-asset portfolio is:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2 \right]^{1/2}$$

2. Perfect positive correlation (i.e., $\rho = 1$): the portfolio standard deviation reduces to the simple weighted average of the individual standard deviations. The portfolio possibilities curve for two perfectly correlated assets is a straight line indicating that there are no benefits from diversifying from a one-asset to a two-asset portfolio if the assets are perfectly correlated.

Perfect negative correlation (i.e., $\rho = -1$): The greatest diversification is achieved when two assets are negatively correlated. The portfolio possibilities curve is two line segments, and it is possible to construct a portfolio with zero standard deviation.

Zero correlation: When the correlation between two assets is zero, the covariance term in the portfolio standard deviation expression is eliminated. The portfolio possibilities curve is non-linear in this case.

Moderate correlation: Most equities are positively correlated (i.e., $0 < \rho < 1$). The portfolio possibilities curve is non-linear in this case.

3. The portfolio possibilities curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.
4. The minimum variance portfolio is the portfolio with the smallest variance among all possible portfolios on a portfolio possibilities curve.
5. The efficient frontier is a plot of the expected return and risk combinations of all efficient portfolios on the portfolio possibilities curve. An efficient portfolio has the highest return for all portfolios with equal volatility and the lowest volatility for all portfolios with equal return.

When short sales are allowed, the efficient frontier expands up and to the right (i.e., higher return and higher volatility portfolio combinations become feasible).

When risk-free lending and borrowing are available, the efficient frontier becomes a straight line. A risk-free asset is the security that has a return known ahead of time, so the variance of the return is zero. The standard deviation of the risk-free asset plus a risky portfolio is:

$$\sigma_C = w_P \sigma_P$$

The equation for the efficient frontier when the risk-asset is available is as follows:

$$E(R_C) = R_F + \left(\frac{E(R_P) - R_F}{\sigma_P} \right) \sigma_C$$

The intercept of this line is equal to the risk-free rate, and the slope is equal to the reward-to-risk ratio for the risky portfolio.

CONCEPT CHECKERS

1. Assume the following information for stocks A and B.
- Expected return on Stock A = 18%.
 - Expected return on Stock B = 23%.
 - Correlation between returns of Stock A and Stock B = 0.10.
 - Standard deviation of returns on Stock A = 40%.
 - Standard deviation of returns on Stock B = 50%.

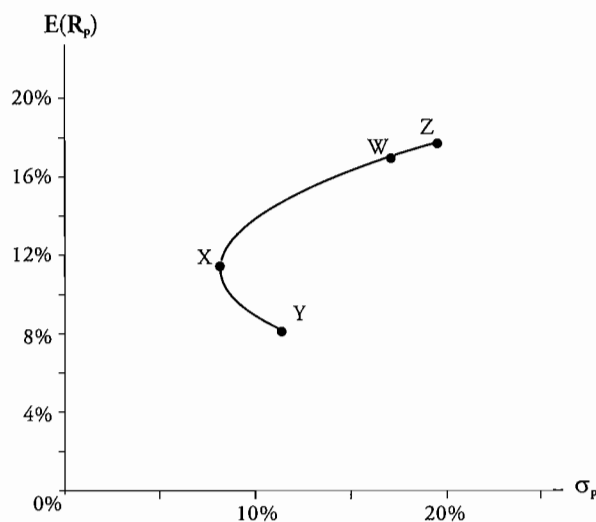
The expected return and standard deviation of an equally weighted portfolio of stocks A and B are closest to:

	<u>Expected return (%)</u>	<u>Standard deviation (%)</u>
A. 20.5		33.54
B. 20.5		11.22
C. 33.5		11.22
D. 33.5		33.54

Use the following data to answer Questions 2 and 3.

Assume the expected return on stocks is 18% (represented by Z in the figure), and the expected return on bonds is 8% (represented by point Y on the graph).

Portfolio Possibilities Curve: Stocks and Bonds



2. The graph shows the portfolio possibilities curve for stocks and bonds. The point on the graph that most likely represents a 90% allocation in stocks and a 10% allocation in bonds is Portfolio:
- A. W.
 - B. X.
 - C. Y.
 - D. Z.

3. The efficient frontier consists of the portfolios between and including:
 - A. X and W.
 - B. Y and Z.
 - C. X and Z.
 - D. Y and X.

4. Which of the following best describes the shape of the portfolio possibilities curve?
 - A. The curve is strictly convex.
 - B. The curve is strictly concave.
 - C. The curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.
 - D. The curve is convex above the minimum variance portfolio and concave below the minimum variance portfolio.

5. When short sales are possible (i.e., there are no short sale restrictions), the efficient frontier is:
 - A. a straight line between the risk-free asset and the market portfolio.
 - B. two line segments, which indicate a negative relationship between short and long positions.
 - C. expanded to include portfolios with higher return and lower volatility.
 - D. expanded to include portfolios with higher return and higher volatility.

CONCEPT CHECKER ANSWERS

1. A $E(R_p) = w_A E(R_A) + w_B E(R_B) = (0.50)(0.18) + (0.50)(0.23) = 0.205 = 20.5\%$

$$\sigma_p = \left[w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \rho_{AB} \sigma_A \sigma_B \right]^{1/2}$$

$$\sigma_p = \left[(0.5)^2 (0.4)^2 + (0.5)^2 (0.5)^2 + 2(0.5)(0.5)(0.1)(0.4)(0.5) \right]^{1/2} = 0.3354 = 33.54\%$$
2. A Since the return to *W* is the nearest to *Z* (stocks), it is logical to assume that point *W* represents an allocation of 90% stocks/10% bonds. The return for *W* is lower than *Z*, but it also represents a reduction in risk.
3. C The *efficient frontier* consists of portfolios that have the maximum expected return for any given level of risk (standard deviation or variance). The efficient frontier starts at the global minimum-variance portfolio and continues above it. Any portfolio below the efficient frontier is dominated by a portfolio on the efficient frontier. This is because efficient portfolios have higher expected returns for the same level of risk.
4. C The portfolio possibilities curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.
5. D When short sales are allowed, the efficient frontier expands up and to the right (i.e., higher return and higher volatility portfolio combinations become feasible). When considering two stocks, by shorting the stock with lower expected return and using the proceeds to increase the investment in the other stock, it is possible to increase portfolio return. This increased return comes at a cost of higher volatility, though.

THE STANDARD CAPITAL ASSET PRICING MODEL

Topic 3

EXAM FOCUS

This topic continues the discussion of the capital market line (CML) and the capital asset pricing model (CAPM). The CAPM requires many assumptions, such as the existence of a risk-free asset and that all investors have the same type of utility function and expectations. The existence of a risk-free asset means the efficient frontier becomes a straight line, which allows for the use of simple expressions to analyze price risk. It is important to have a firm grasp on the calculation methodology of the CAPM.

THE CAPITAL ASSET PRICING MODEL (CAPM)

AIM 3.2: Describe the assumptions underlying the CAPM.

The **capital asset pricing model (CAPM)**, derived by Sharpe, Lintner, and Mossin, is one of the most celebrated models in all of finance. The model describes the relationship we should expect to see between risk and return for individual assets. Specifically, the CAPM provides a way to calculate an asset's expected return (or "required" return) based on its level of systematic (or market-related) risk, as measured by the asset's beta.

CAPM Assumptions

In the derivation of any economic or scientific model, simplifying assumptions regarding the market, which the model represents, must be made. The CAPM has a number of underlying assumptions:

1. Investors face no transaction costs when trading assets. This assumption simplifies the computation of returns. If transaction costs were considered, returns would be a function of transaction costs, which would then have to be estimated.
2. Assets are infinitely divisible. It is possible to hold fractional shares.
3. There are no taxes; therefore, investors are indifferent between capital gains and income or dividends.
4. Investors are price takers whose individual buy and sell decisions have no effect on asset prices. The market for assets is perfectly competitive.
5. Investors' utility functions are based solely on expected portfolio return and risk. This assumption provides a framework for how investors make investment decisions.
6. Unlimited short-selling is allowed. Investors can sell an unlimited number of shares of an asset short.

7. Investors can borrow and lend unlimited amounts at the risk-free rate.
8. Investors are only concerned about returns and risk over a single period, and the single period is the same for all investors.
9. All investors have the same forecasts of expected returns, variances, and covariances. This is known as homogeneous expectations.
10. All assets are marketable, including human capital.

THE CAPITAL MARKET LINE (CML)

AIM 3.3: Describe the capital market line.

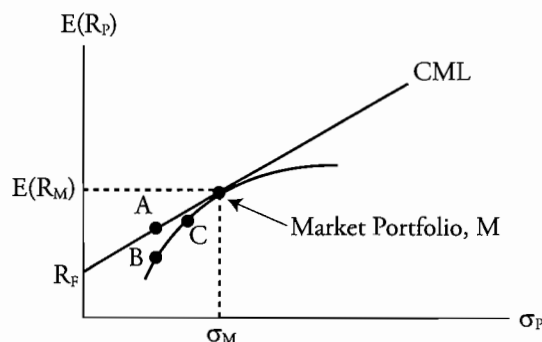
In the presence of riskless lending and borrowing, the efficient frontier transforms from a curve to a line tangent to the previous curve. Investors will choose to invest in some combination of their tangency portfolio and the risk-free asset. Assuming investors have identical expectations regarding expected returns, standard deviations, and correlations of all assets, there will be only one tangency line, which is referred to as the **capital market line (CML)**.

Under the assumptions of the CML, all investors agree on the exact composition of the optimal risky portfolio. This universally agreed upon optimal risky portfolio is called the **market portfolio, M**, and it is defined as the portfolio of all marketable assets weighted in proportion to their relative market values. For instance, if the market value of Asset X is \$1 billion, and the market value of all traded assets is \$100 billion, then the weight allocated to Asset X in the market portfolio equals 1%.

The key conclusion of the CML can be summarized as follows: *All investors will make optimal investment decisions by allocating between the risk-free asset and the market portfolio.*

Figure 1 provides a graph of the CML.

Figure 1: The Capital Market Line



The equation for the CML is:

$$E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_P$$

The slope of the CML is often called the *market price of risk* and equals the reward-to-risk ratio (or Sharpe ratio) for the market portfolio. This is calculated as:

$$\frac{E(R_M) - R_F}{\sigma_M}$$



Professor's Note: We will examine the calculation of risk-adjusted return measures, such as the Sharpe ratio, in Topic 6.

The CML is useful for computing the expected return for an efficient (diversified) portfolio; however, it cannot compute the expected return for inefficient portfolios or individual securities. The CAPM must be used to compute the expected return for any inefficient portfolio or individual security.

DERIVING THE CAPM

AIM 3.1: Understand the derivation and components of the CAPM.

A Straightforward Derivation

The procedure used to derive the equation for the capital asset pricing model requires an understanding of the characteristics of expected return, beta, the risk-free rate, and the security market line. The following steps illustrate how the CAPM is derived. The end result will be an equation where the expected return on a single security or portfolio of securities is equal to:

$$R_F + \text{Beta}_i [E(R_M) - R_F]$$

The first step in the derivation is to recognize that beta identifies the appropriate level of risk for which an investor should be compensated. An important concept in finance is that, as a portfolio becomes more diversified, idiosyncratic risk (i.e., unsystematic risk or asset-specific risk) in the portfolio becomes less of an issue as only systematic risk remains.

Professor's Note: Starting with the formula for portfolio variance, and assuming n equally-weighted assets (e.g., each $w = 1/n$), it is possible to show that the portfolio variance for an equally-weighted portfolio is:

$$\sigma_P^2 = \frac{1}{n} \overline{\sigma_i^2} + \frac{n-1}{n} \overline{Cov}$$

where:

$\overline{\sigma_i^2}$ = average variance of all assets in the portfolio

\overline{Cov} = average covariance of all pairings of assets in the portfolio



Note that the equally-weighted portfolio variance equals the sum of two components (unsystematic risk: the variance term and systematic risk: the covariance term), each of which is affected by the size of the portfolio:

- $(1/n) \times \overline{\sigma_i^2}$ gets closer to zero as n gets larger because $1/n$ approaches zero.
- $[(n-1)/n] \times \overline{Cov}$ gets closer to the average covariance as n gets larger because $(n-1)/n$ approaches 1.

Therefore, the following important result emerges: The variance of an equally-weighted portfolio approaches the average covariance as n gets large.

Since diversification is costless and systematic risk is the only remaining risk in a diversified portfolio, an investor should only be compensated for systematic risk (or beta) exposure. Therefore, all assets with the same beta should earn the same return.

The next step in the derivation is to recognize that expected return is a linear function of beta. Since portfolio beta is the weighted average of the individual betas and expected portfolio return is a weighted average of the individual expected returns, the portfolio expected return is a linear function of beta.

$$E(R_P) = a + m \times \beta_P$$

where:

$$\beta_P = \frac{Cov_{P,M}}{\sigma_M^2}$$

$Cov_{P,M}$ = covariance between the returns for Stock P and the market portfolio

σ_M^2 = variance of the returns on the market portfolio

Professor's Note: To show that portfolio return is a linear function of beta, start with the functions for expected portfolio return and portfolio beta.

$$\begin{aligned} E(R_P) &= w_1 E(R_1) + (1 - w_1) E(R_2) \\ \beta_P &= w_1 \beta_1 + (1 - w_1) \beta_2 \end{aligned}$$

Solve w_1 in the portfolio beta equation:

$$w_1 = \frac{\beta_P - \beta_2}{\beta_1 - \beta_2}$$



Substitute w_1 into the portfolio expected return equation:

$$\begin{aligned} E(R_P) &= \frac{\beta_P - \beta_2}{\beta_1 - \beta_2} E(R_1) + \left[1 - \frac{\beta_P - \beta_2}{\beta_1 - \beta_2} \right] E(R_2) \\ E(R_P) &= \left[E(R_2) - \frac{\beta_2 [E(R_1) - E(R_2)]}{\beta_1 - \beta_2} \right] + \beta_P \frac{[E(R_1) - E(R_2)]}{\beta_1 - \beta_2} \\ E(R_P) &= a + m \beta_P \end{aligned}$$

where:

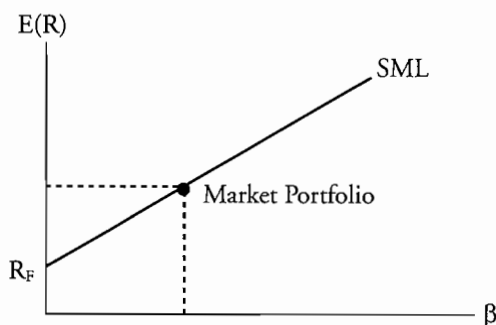
$$\begin{aligned} a &= E(R_2) - \frac{\beta_2 [E(R_1) - E(R_2)]}{\beta_1 - \beta_2} \\ m &= \frac{[E(R_1) - E(R_2)]}{\beta_1 - \beta_2} \end{aligned}$$

Assets with equivalent betas should earn the same return because arbitrage will prevent assets with the same risk from earning different returns. So, if $\beta_i = \beta_P$ and $E(R_i) = E(R_P)$, then we can express the expected return for asset i as a linear function of its beta:

$$E(R_i) = a + m \times \beta_i$$

As shown in Figure 2, this equation plots a straight line, known as the security market line (SML) with an intercept of a and slope of m . Thus, the SML is a graphical representation of the CAPM.

Figure 2: The Security Market Line



The final step in this derivation is to find two points on the SML and solve for the CAPM. To solve for the equation of a line (which is known as identifying the line), we need to know two points on the line. Fortunately, we do know two of the points on this line: the risk-free asset and the market portfolio. Since it has no risk, the risk-free asset has a beta of zero; therefore, the intercept of the SML is R_F and our first point is $(0, R_F)$. The market portfolio has a beta of one, so the second point is $[1, E(R_M)]$. With these two points, we can find the slope of the line, m :

$$E(R_i) = a + m \times \beta_i$$

$$E(R_M) = R_F + m \times 1$$

$$m = E(R_M) - R_F$$

Professor's Note: It is relatively straightforward to see that the beta of the market is one. The covariance of the market with itself is equal to the variance of the market. Therefore, solving for market beta, we get:



$$\beta_M = \frac{Cov_{M,M}}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

With information on both the intercept (a) and the slope (m), we are now able to display the well-known capital asset pricing model:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

CALCULATING EXPECTED RETURN USING CAPM

AIM 3.4: Use the CAPM to calculate the expected return on an asset.

Example: Expected return on a stock

Assume you are assigned the task of evaluating the stock of Sky-Air, Inc. To evaluate the stock, you calculate its required return using the CAPM. The following information is available:

expected market risk premium	5%
risk-free rate	4%
Sky-Air beta	1.5

Using CAPM, calculate and interpret the expected return for Sky-Air.

Answer:

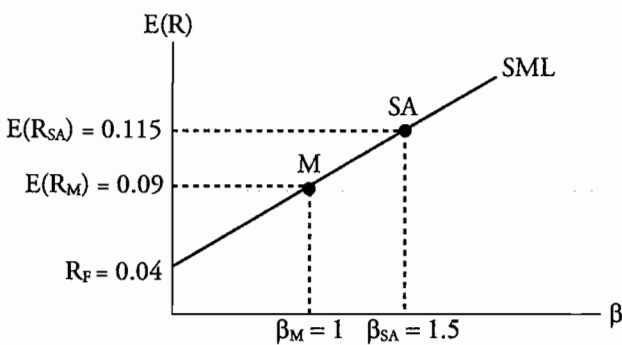
The expected return for Sky-Air is:

$$E(R_{SA}) = 0.04 + 1.5(0.05) = 0.115 = 11.5\%$$

Recall that the market risk premium is the expected market return minus the risk-free rate. The CAPM return can be viewed as the minimum return that investors should be willing to accept (i.e., the required rate of return), commensurate with the risk associated with the asset. For example, if investors predict that the return will exceed 11.5%, then they should buy Sky-Air stock. However, if investors predict that the return will be less than 11.5%, then they should sell Sky-Air stock (or short the stock).

Figure 3 illustrates the required return for Sky-Air on the SML.

Figure 3: Sky-Air Plotted on the Security Market Line



In the previous example, we calculated the required rate of return, which always lies on the security market line. If through the valuation of an asset an analyst determines that the expected return is different from the required rate of return implied by CAPM, then the security may be mispriced according to rational expectations. A mispriced security would not lie on the security market line. In general:

- An overvalued security would have a required rate of return (computed by CAPM) that is higher than its expected return (computed by the analyst's valuation). An overvalued security would lie below the security market line.
- An undervalued security would have a required rate of return (computed by CAPM) that is lower than its expected return (computed by the analyst's valuation). An undervalued security would lie above the security market line.

In addition to computing the required or expected return for an individual asset, it is possible to solve for the expected return on the market and/or the market risk premium given the risk-free rate, expected return on an asset, and the systematic risk for that asset.

Example: Using CAPM to calculate the expected market return

A stock has a beta of 0.75 and an expected return of 13%. The risk-free rate is 4%. Calculate the market risk premium and the expected return on the market portfolio.

Answer:

According to CAPM: $0.13 = 0.04 + 0.75[E(R_M) - R_F]$.

Therefore, the market risk premium is equal to: $[E(R_M) - R_F] = 0.12 = 12\%$.

The expected return on the market is calculated as: $[E(R_M) - 0.04] = 0.12$, or $E(R_M) = 0.16 = 16\%$.

KEY CONCEPTS

1. The capital asset pricing model (CAPM), derived by Sharpe, Lintner, and Mossin, expresses the expected return for an asset as a function of the asset's level of systematic risk (measured by beta), the risk-free rate, and the market risk premium (the expected return of the market minus the risk-free rate). There are several assumptions underlying the CAPM.
 - Investors face no transaction costs.
 - Assets are infinitely divisible.
 - There are no taxes.
 - Investors are price takers whose individual buy and sell decisions have no effect on asset prices.
 - Investors' utility functions are based solely on expected portfolio return and risk.
 - Unlimited short-selling is allowed.
 - Investors are only concerned about returns and risk over a single period, and the single period is the same for all investors.
 - All investors have the same forecasts of expected returns, variances, and covariances.
 - All assets are marketable.
2. There are three major steps in deriving the CAPM:
 1. Recognize that since investors are only compensated for bearing systematic risk, beta is the appropriate measure of risk.
 2. By knowing that portfolio expected return is a weighted average of individual expected returns and portfolio beta is a weighted average of the individual betas, we can show that portfolio return is a linear function of portfolio beta. Since arbitrage prevents mispricing of assets relative to systematic risk (beta), an individual asset's expected return is a linear function of its beta.
 3. Use the risk-free asset and the market portfolio, which are two points on the security market line, to solve for the intercept and slope of the CAPM. The equation for CAPM is:

$$E(R_i) = R_F + [E(R_M) - R_F]\beta_i$$

3. The capital market line (CML) expresses the expected return of a portfolio as a linear function of its standard deviation, the market portfolio's return and standard deviation, and the risk-free rate.

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_C$$

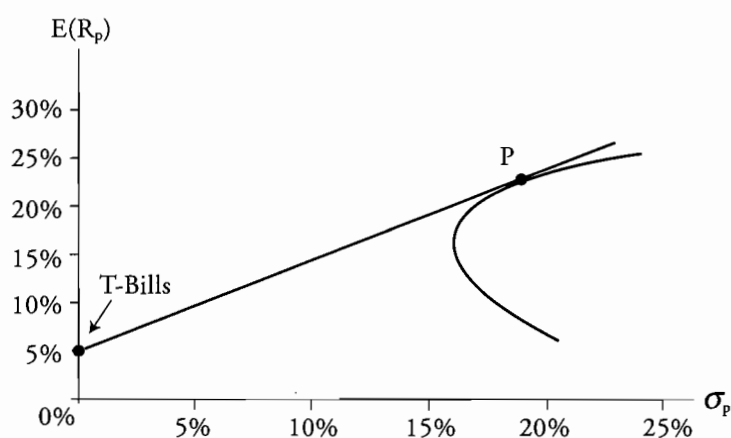
4. The expected return for an asset can be computed using the CAPM given the risk-free rate, the market risk premium, and an asset's systematic risk.

CONCEPT CHECKERS

1. Which of the following statements is most likely an assumption of the capital asset pricing model (CAPM)?
 - A. Investors only face capital gains taxes.
 - B. Investors' actions affect the prices of assets.
 - C. Transaction costs are constant across all assets.
 - D. All assets including human capital are marketable.

Use the following graph to answer Question 2.

Mean-Variance Analysis



2. Portfolio P in the mean variance analysis represents the tangency point between the capital market line and the portfolio possibilities curve. In this analysis, the market price of risk would be the:
 - A. standard deviation of Portfolio P.
 - B. expected return on the minimum-variance portfolio.
 - C. slope of the line connecting T-bills and Portfolio P.
 - D. point at which the straight line intersects the expected return axis.

3. At a recent analyst meeting at Invest Forum, analysts Michelle White and Ted Jones discussed the use of the capital market line (CML). White states that the CML assumes that investors hold two portfolios: 1) a risky portfolio of all assets weighted according to their relative market value capitalizations; and 2) the risk-free asset. Jones states that the CML is useful in determining the required rate of return for individual securities.

Are the statements of White and Jones correct?

 - A. Only Jones's statement is correct.
 - B. Only White's statement is correct.
 - C. Both statements are correct.
 - D. Neither statement is correct.

4. Patricia Franklin makes buy and sell stock recommendations using the capital asset pricing model. Franklin has derived the following information for the broad market and for the stock of the CostSave Company (CS):

- Expected market risk premium 8%
- Risk-free rate 5%
- Historical beta for CostSave 1.50

Franklin believes that historical betas do not provide good forecasts of future beta, and therefore uses the following formula to forecast beta:

$$\text{forecasted beta} = 0.80 + 0.20 \times \text{historical beta}$$

After conducting a thorough examination of market trends and the CS financial statements, Franklin predicts that the CS return will equal 10%. Franklin should derive the following required return for CS along with the following valuation decision (undervalued or overvalued):

<u>Valuation</u>	<u>CAPM required return</u>
A. overvalued	8.3%
B. overvalued	13.8%
C. undervalued	8.3%
D. undervalued	13.8%

5. Albert Dreiden wants to estimate the expected return on the market. He believes that the stock of the Hobart Materials Company is fairly valued, and gathers the following information:

- Expected return for Hobart 7.50%
- Risk-free rate 4.50%
- Beta for Hobart 0.80

Based on this information, the estimated expected return for the market portfolio is closest to:

- A. 3.00%.
- B. 3.75%.
- C. 6.90%.
- D. 8.25%.

CONCEPT CHECKER ANSWERS

1. **D** The capital asset pricing model (CAPM) assumes that all assets including human capital are marketable. Additionally, CAPM assumes no taxes, no transaction costs, and that investor actions do not affect market prices.
2. **C** The CML is the line connecting T-bills and Portfolio P. The market price of risk is the slope of the CML. Had risk been measured on the graph with beta, the graph would represent the SML. The market price of risk would still be the slope of the line.
3. **B** The CML assumes all investors have identical expectations and all use mean-variance analysis, implying that they all identify the same risky tangency portfolio (the “market portfolio”) and combine that risky portfolio with the risk-free asset when creating their portfolios. Because all investors hold the same risky portfolio, the weight on each asset must be equal to the proportion of its market value to the market value of the entire portfolio. Therefore, White is correct. The CML is useful for determining the rate of return for efficient portfolios, but it cannot be used to determine the required rate of return for inefficient portfolios or individual securities. The capital asset pricing model (CAPM) is used to determine the required rate of return for inefficient portfolios and individual securities. Therefore, Jones is incorrect.

4. **B** The CAPM equation is:

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$$

Franklin forecasts the beta for CostSave as follows:

$$\text{beta forecast} = 0.80 + 0.20 (\text{historical beta})$$

$$\text{beta forecast} = 0.80 + 0.20(1.50) = 1.10$$

The CAPM required return for CostSave is:

$$0.05 + 1.1(0.08) = 13.8\%$$

Note that the market premium, $E(R_M) - R_F$, is provided in the question (8%).

Franklin should decide that the stock is overvalued because she forecasts that the CostSave return will equal only 10%, whereas the required return (minimum acceptable return) is 13.8%.

5. **D** The CAPM equation is:

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$$

Using the given information, we can solve for the expected return for the market portfolio as follows:

$$7.50\% = 4.50\% + 0.80[E(R_M) - 4.50\%]$$

$$E(R_M) = (7.50\% - 4.50\%) / 0.80 + 4.50\% = 8.25\%$$

Based on the information given and using the CAPM, the expected return on the market is 8.25%.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

NONSTANDARD FORMS OF CAPITAL ASSET PRICING MODELS

Topic 4

EXAM FOCUS

This topic explores various deviations from the simple form of the CAPM; however, the simple form CAPM still endures rather well when relaxing assumptions. Some changes in assumptions lead to notable changes in the equilibrium relationship, but many conclusions drawn from the original CAPM still hold. However, this is only the case when looking at the changes in assumptions one at a time. If multiple assumptions change during the same testing period, the departure from CAPM will be more significant regarding what is predicted by the CAPM and what is actually observed in practice.

RELAXING ASSUMPTIONS OF THE CAPITAL ASSET PRICING MODEL (CAPM)

AIM 4.1: Describe the impact on the CAPM of the following:

- Short sales disallowed
 - Riskless lending and borrowing
 - Personal taxes
 - Nonmarketable assets
 - Heterogeneous expectations
 - Non-price-taking behavior
-

From previous topics, we know that the CAPM requires several restrictive assumptions in order to hold. For this AIM, we will analyze the effects of relaxing individual assumptions stated by the standard CAPM. The impact on the CAPM when relaxing each assumption is as follows:

Short Sales Disallowed

CAPM assumes unlimited short sales are permitted. This assumption is not necessary, since within the CAPM framework, investors hold the market portfolio in equilibrium; a situation in which no investor would sell a security short. Therefore, the short sales disallowed assumption has no impact on CAPM.

Riskless Lending and Borrowing

CAPM assumes that investors can borrow and lend unlimited amounts at the risk-free rate.

Case 1: No risk-free rate at which to lend or borrow

In this case, a zero beta portfolio (i.e., a risk-free portfolio) is used in place of the risk-free asset to derive a zero beta CAPM. In the derivation of CAPM, two points on the SML were used to find the equation's intercept and slope of the CAPM. Recall that in order to derive CAPM we used the risk-free asset and the market portfolio. If the risk-free rate is unavailable, a zero beta portfolio can instead be used as follows:

$$E(R_i) = E(R_Z) + [E(R_M) - E(R_Z)]\beta_i$$

where:

$E(R_Z)$ = expected return for a zero beta portfolio

Since there are actually multiple zero beta portfolios, the correct zero beta portfolio will be the one with the smallest total risk. Note that the covariance (and beta) of the zero beta portfolio and the market portfolio is zero. Thus, the zero beta portfolio will lie inside of the efficient frontier.

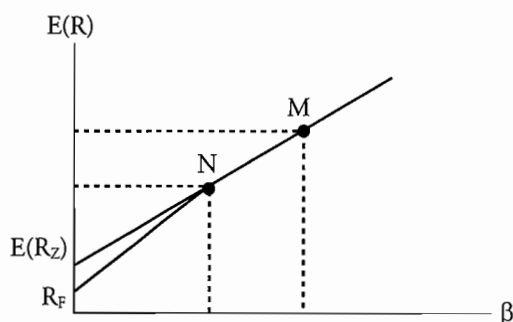
The result of the zero beta version of CAPM is that investors still hold two funds; however, instead of the risk-free asset, they hold some combination of the zero beta portfolio and the market portfolio. Since the zero beta portfolio is inefficient, no investor will hold only this portfolio.

Case 2: Risk-free rate at which to lend, but no risk-free rate at which to borrow

Another important case to consider (that is more realistic) is the case of riskless lending, but no riskless borrowing. Risk-free assets do exist (e.g., U.S. Treasury bills), so riskless lending is feasible. However, riskless borrowing is not available to most investors.

Under the assumption of only a risk-free lending rate, the SML will change in shape to look more like the graph in Figure 1. All risky portfolios that are formed as a combination of the market portfolio and the zero beta portfolio have a required return given on the line $E(R_Z)M$. Efficient portfolios that contain the risk-free asset should earn a return along the line segments R_FN and NM , where N is a combination of the market portfolio and the zero beta portfolio. The result in this case is that investors will hold three funds: the risk-free asset, the market portfolio, and the zero beta portfolio.

Figure 1: SML in the case of riskless lending but no riskless borrowing



The important concepts from these two cases are:

1. In aggregate, the market portfolio holds; however, individual investors will hold different portfolios of risky assets (i.e., combining the market portfolio and zero beta portfolio results in different risky portfolios).
2. The first case results in a two-mutual-fund theorem, while the second case results in a three-fund theorem.
3. In the first case, the SML is a straight line; however, the intercept is now the expected return on the zero beta portfolio, and the slope is the difference between the expected market return and the expected zero beta return. The second case results in a straight line for risky assets and two-line segments for efficient portfolios containing the risk-free asset.

Personal Taxes

Another limiting assumption of the CAPM is that it ignores personal taxes. The presumption is that investors are indifferent toward receiving income in the form of capital gains or dividends. This is not very realistic, since investors should judge portfolio performance on an after-tax basis, and the tax rates differ between taxes on capital gains versus dividends. The formula demonstrating a proper equilibrium relationship assuming different tax rates on income and capital gains is as follows:

$$E(R_P) = R_F + \beta_i [(E(R_M) - R_F) - \tau(\delta_M - R_F)] + \tau(\delta_i - R_F)$$

where:

δ_M = dividend yield of market portfolio

δ_i = dividend yield for stock i

τ = tax factor that measures market tax rates on both capital gains and income

Previously, the only variable connected with an individual security that affected expected return was beta. However, from the previous formula, we can see that the dividend yield also affects expected return in the presence of taxes. With the CAPM, the equilibrium relationship was represented by a straight line; however, this revised equilibrium relationship will be a plane.

An investor with a diversified portfolio will have a portfolio similar to the market portfolio, but it will be tilted in the direction whereby that investor has a competitive advantage. It will be a function of the difference in tax rates on dividends versus capital gains compared with the investor's tax bracket. When assuming a higher tax rate on dividends versus capital gains, the greater the investor's return that is received in the form of dividends, the more taxes that are paid, which requires more pretax return.

Nonmarketable Assets

CAPM assumes all assets are easily marketable. A more realistic assumption is that every investor has nonmarketable assets in their portfolio. These include assets that may be technically "marketable," but the investor considers them a fixed portion of the portfolio.

There may be various reasons for this, including large transaction costs, but non-financial factors may also be present.

Accepting the premise that investors have marketable and nonmarketable assets, the following formula results as the equilibrium return on all assets:

$$E(R_J) = R_F + \frac{E(R_M) - R_F}{\sigma_M^2 + \frac{P_H}{P_M} \text{cov}(R_M, R_H)} \times \left[\text{cov}(R_J, R_M) + \frac{P_H}{P_M} \text{cov}(R_J, R_H) \right]$$

where:

R_H = one period return rate on nonmarketable assets

P_H = total value of all nonmarketable assets

P_M = total value of all marketable assets

Revising the simple CAPM to incorporate nonmarketable assets creates a new equilibrium relationship:

$$\frac{E(R_M) - R_F}{\sigma_M^2 + \frac{P_H}{P_M} \text{cov}(R_M, R_H)}$$

Given that all portfolios will contain a nonmarketable portion of assets, the revised equilibrium equation will assist us in examining this “missing asset” problem. The presumption is that all portfolios have some assets that are “left out.”

Heterogeneous Expectations

There have been several attempts to find a general equilibrium solution assuming that investors have heterogeneous expectations. Equilibrium can still be viewed in terms of expected returns, covariances, and variances in this case. However, the factors are now sophisticated weighted-averages of various estimates held by different individuals. Weightings vary because investor utility functions vary: investors have different marginal rates of substitution between expected return and variance. To determine the proper risk-return trade-offs, prices are required. The end result is that it is difficult to determine a solution for the heterogeneous expectation problem.

It appears that beta is an adequate risk measure and that equilibrium models lead to a linear relationship between expected return and beta, consistent with more simple forms of CAPM.

Non-Price-Taking Behavior

A general assumption has been that all individuals are “price takers,” meaning that they disregard the effect that their buying and selling behavior have on the equilibrium price of securities, and thus on their individual optimal portfolio composition. However, what about large investors who may believe their behavior *does* affect price? Studies have shown that the “price affector” will be “less of a risk avoider” (i.e., hold less of the risk-free asset) than compared to a price affector who does not recognize that his actions affect price. The

price affector's actions increase utility, but the end result still leads to the simple form of the CAPM, with the market price of risk being lower than it would be if all investors had simply been "price takers."

MULTI-PERIOD CAPITAL ASSET PRICING MODELS

AIM 4.2: Describe the following multi-period versions of CAPM:

- Consumption-oriented CAPM
 - CAPM including inflation
 - Multi-beta CAPM
-

The CAPM we have studied thus far assumes that investors make their investment decisions at a single point in time. More realistically, we should incorporate investment decisions over the lifetime of the investor. Fortunately, many of the conditions that are appropriate for the single-period CAPM are also appropriate when evaluating the CAPM in a multi-period setting. However, the process of maximizing an investor's utility will change over time, so it is helpful to examine the CAPM given differing assumptions. Three versions of the multi-period CAPM are: (1) consumption-oriented CAPM, (2) CAPM including inflation, and (3) multi-beta CAPM

Consumption-Oriented CAPM

This approach incorporates a different set of assumptions in defining equilibrium in the capital markets:

- Investors maximize a multi-period utility function for consumption over a lifetime.
- Investors have homogeneous beliefs regarding assets' return characteristics.
- The number of investors is fixed.
- There is a single consumption good.
- There is a capital market that permits investors to achieve a consumption pattern under which they cannot fare better by engaging in additional trades.

Under these different assumptions, return on assets should be linearly related to the aggregate consumption growth rate. In addition, the residuals from this linear relationship are not correlated with the aggregate consumption's growth rate; they have a zero mean, and are not correlated with one another.

The resulting equilibrium condition can be shown as follows:

$$E(R_i) = E(R_Z) + \gamma_1 \beta_1$$

where:

γ_1 = market price of consumption beta

$E(R_Z)$ = expected portfolio return assuming zero consumption beta

Similar to CAPM's simple form, growth rate of per capita consumption replaces the market portfolio's return as the factor affecting the time series of returns, or equilibrium returns.

CAPM Including Inflation

A 1976 study demonstrated that the equilibrium relationship for the expected return on any asset, under uncertain inflationary conditions, looks to be similar to the CAPM simple form. However, market price of risk and asset risk are both modified when including inflation. Four primary differences were found between standard CAPM and multi-period CAPM:

1. If the correlation between the market return and the inflation rate is positive, market risk is greater than it is under the standard CAPM.
2. An asset's risk is a function of not only its covariance with the market, but also its covariance with the inflation rate.
3. If the rate of return on an asset is positively correlated with the inflation rate, the simple form CAPM formula overstates the asset's risk.
4. Simple form CAPM will understate (overstate) any assets equilibrium return rate if the correlation of the asset's return with the inflation rate is less than (greater than) the product of the correlation of an asset's return with market return, and the correlation between market return and inflation rate.

Multi-Beta CAPM

The simplest form of a multi-beta CAPM is an inflation model. A new term is added to the standard CAPM—it is the product of a revised beta (sensitivity of any security to the portfolio that is selected as an inflation risk hedge) and inflation risk's price. This formula is as follows:

$$E(R_i) - R_F = \beta_{iM}[E(R_M) - R_F] + \beta_{iI}[E(R_I) - R_F]$$

More generally, the multi-beta CAPM demonstrates that a security's expected return should be related to the security's sensitivity to a set of influences, such as: future labor income, pricing on consumable goods, investment opportunities, etc. In this case, the multi-beta CAPM becomes:

$$E(R_i) - R_F = \beta_{iM}[E(R_M) - R_F] + \beta_{iI1}[E(R_{I1}) - R_F] + \beta_{iI2}[E(R_{I2}) - R_F] + \dots$$

This approach permits investors to have unique hedges against specific risks with which they are concerned. In addition to market risk, investors may be concerned with risks such as: default risk, term structure risk, deflation risk, and profit risk. These risks are important for investors; however, each individual investor will want to incorporate the most relevant factors to their situations when analyzing their own securities and/or portfolios.

KEY CONCEPTS

1. CAPM is effective when it comes to describing equilibrium returns on a macro level, but it is not necessarily accurate when examining individual investors' behavior. Alternative models to CAPM are examined by including real-world influences, such as the existence of short sales, no risk-free borrowing rate, the existence of taxes, inclusion of nonmarketable assets, investor heterogeneous expectations, and the inclusion of "price affectors." The end result is that many of CAPM's original conclusions still hold, even after relaxing many of the previous assumptions.
2. Multi-period versions of CAPM include consumption-oriented CAPM, CAPM including inflation, and multi-beta CAPM. We find that growth rate of per capita consumption does affect the equilibrium returns. Also, analyzing CAPM, when including inflation, results in an equilibrium relationship for the expected return on any asset that looks to be similar to the CAPM simple form; however, market price of risk and asset risk are both modified. Lastly, it was determined that a multi-beta CAPM would allow an investor to have unique hedges against specific risks that are of most concern to the investor.

CONCEPT CHECKERS

1. One of the assumptions of the CAPM is that it ignores taxes. When considering taxes, and the difference in tax rates between dividends and capital gains, expected return would be affected by:
 - A. beta and dividend yield.
 - B. beta only.
 - C. dividend yield only.
 - D. alpha and beta.

2. Consider a consumption-oriented CAPM and assume that investors maximize a multi-period utility function for consumption over a lifetime. In this scenario, return on assets should be:
 - A. uncorrelated with aggregate consumption growth rate.
 - B. linearly related to the aggregate consumption growth rate.
 - C. negatively correlated to the aggregate consumption growth rate.
 - D. linearly related to the portfolio's market return.

3. Given the following equilibrium formula, $E(R_i) = 0.02 + 0.08\beta_i$, and assuming that the zero beta model is applied when relaxing the CAPM's riskless borrowing and lending assumption, what would be the zero beta return and the market return?

<u>Zero beta return</u>	<u>Market return</u>
A. 2%	6%
B. 8%	10%
C. 2%	10%
D. 8%	6%

4. If personal taxes are incorporated into the standard CAPM equation, what will the tax factor be given the following expression: $E(R_i) = 0.02 + 0.078\beta_i + 0.03\delta_i$? Assume that the dividend yield on the market portfolio is 2.6%, the dividend yield for the stock in question is 2.2%, and the market risk premium is equal to 8%.
 - A. 33%.
 - B. 3%.
 - C. 10.8%.
 - D. 5%.

5. When inflation is added to the CAPM measure, some differences arise between the multi-period CAPM and the single-period CAPM. Which of the following statements does not accurately portray one of those differences?
 - A. If the correlation between the market return and the inflation rate is negative, market risk is greater in the multi-period CAPM than it is under the standard CAPM.
 - B. An asset's risk is a function of not only its covariance with the market, but also its covariance with the inflation rate.
 - C. If the rate of return on an asset is positively correlated with the inflation rate, the simple form CAPM formula overstates the asset's risk.
 - D. Simple form CAPM will overstate any assets equilibrium return rate if the correlation of the asset's return with the inflation rate is greater than the product of the correlation of an asset's return with market return, and the correlation between market return and inflation rate.

CONCEPT CHECKER ANSWERS

1. **A** Beta and dividend yield will affect the expected return. Under the simple version of CAPM, the only variable affecting expected return was beta. When taxes are considered, dividend yield also affects expected return. Investors who are in a high tax bracket prefer capital gains over dividends.
2. **B** Using the revised capital market equilibrium assumptions, the growth rate of per capita consumption replaces the market portfolio's return as the factor that affects the time series of returns, or equilibrium returns.
3. **C** The expected return for a zero beta portfolio is equal to 2%. For the market return, we know that the market risk premium is equal to 8%, so the market return will be 10% (i.e., a 10% market return minus a 2% zero beta return will equal the 8% market risk premium).
4. **A** With the introduction of personal taxes, CAPM becomes:

$$E(R_i) = R_F + \beta_i [E(R_M) - R_F - \tau(\delta_M - R_F)] + \tau(\delta_i - R_F)$$

$$E(R_i) = 0.02 + \beta_i [0.08 - \tau(0.026 - 0.02)] + \tau(0.022 - 0.02)$$

$$E(R_i) = 0.02 + 0.078\beta_i + 0.03\delta_i$$

Since we know the dividend yield for stock i and the risk-free rate, we can back into the tax factor, which measures the market tax rate on capital gains and income.

$$0.03\delta_i = 0.03 \times 0.022 = 0.00066 = \tau(0.022 - 0.02); \tau = 33\%$$
5. **A** If the correlation between the market return and the inflation rate is *positive*, market risk is greater than it is under the standard CAPM.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

THE ARBITRAGE PRICING MODEL

APT—A NEW APPROACH TO EXPLAINING ASSET PRICES

Topic 5

EXAM FOCUS

This topic introduces arbitrage pricing theory (APT), which is an equilibrium asset-pricing model with less stringent assumptions than the CAPM. You should have an understanding of the assumptions underlying APT and be able to compute the expected return for an asset given inputs into the APT model. Additionally, you should recognize why the CAPM is a special case of the APT, and you should be able to explain how APT can be applied to passive and active portfolio management.

ARBITRAGE PRICING THEORY (APT)

AIM 5.1: Describe the APT and the assumptions underlying it.

AIM 5.2: Use the APT to calculate the expected return on an asset.

Arbitrage pricing theory (APT) assumes that:

- *Returns are derived from a multifactor model.* Unfortunately, the APT provides little practical guidance for the identification of the risk factors in the model. The lack of clarity for which risk factors should apply is a major weakness of the APT.
- *Unsystematic risk can be completely diversified away.* This implies that unsystematic risk is not priced (i.e., has a zero risk premium).
- *No arbitrage opportunities exist.* An arbitrage opportunity is defined as an investment opportunity that bears no risk, no cost, and yet provides a profit. This assumption implies that investors will undertake infinitely large positions (long and short) to exploit any perceived mispricing, causing asset prices to adjust immediately to their equilibrium values.

The APT Equation

APT assumes that returns can be modeled with a multifactor regression model of the following form:

Equation 1

$$R_n = \alpha_n + \beta_{n,1} \times F_1 + \dots + \beta_{n,k} \times F_k + u_n$$

where:

- R_n = return for stock n
- α_n = expected return when factor exposures are all zero
- $\beta_{n,k}$ = k th factor exposure for stock n
- F_k = factor k
- u_n = idiosyncratic return for stock n

Each F represents an index or macroeconomic factor that drives stock returns. Each β represents the sensitivity (also called factor “loading”) of Portfolio n to each risk factor. Unlike the CAPM, the APT does *not* require that one of the risk factors is the market portfolio. This is a major advantage of the arbitrage pricing model.

Given the model for returns (Equation 1), the expected return for a stock is calculated using the APT formula as follows:

Equation 2

$$E(R_n) = \lambda_0 + \lambda_1 \times \beta_{n,1} + \dots + \lambda_k \times \beta_{n,k}$$

where:

- λ_0 = R_F
- λ_k = expected risk premium associated with each risk factor

Each λ_j stands for the expected risk premium associated with its corresponding risk factor. Remember that a risk premium is the difference between the expected return and the risk-free rate. It is the extra expected return from taking on more risk.

The rationale for APT implies that if you had a model of Equation 1’s form, and expected returns do not follow the form of Equation 2, there would be an arbitrage opportunity. Given an arbitrage situation, an investor could create an investment portfolio, which generates a positive return at no expense.



Professor’s Note: In an empirical or practitioner setting, Equation 1 would be estimated through time series regressions of each portfolio’s return on to the factors, and the estimated β s would be obtained. Then, Equation 2 would be estimated through a cross-sectional regression of the portfolio returns on to the β s that were estimated in the first set of regressions.

Calculating Expected Returns Using APT

Example: Calculating expected returns from the arbitrage pricing model

An investment firm, Invest Fund, employs a two-factor APT model. The risk-free rate equals 5%. Determine the expected return for Invest Fund using the following data:

	<u>Factor 1</u>	<u>Factor 2</u>
Invest Fund factor betas	1.50	2.00
Factor risk premiums	0.0300	0.0125

Answer:

Using the two-factor APT model, the expected return for the Invest Fund (IF) equals:

$$E(R_{IF}) = 0.05 + 1.5(0.03) + 2(0.0125) = 0.12 = 12\%$$

A Simple Proof of the APT

The process of arbitrage ensures that APT holds. To better understand this process, consider the following example.

Example: Exploiting an arbitrage opportunity

Assume your investment firm uses a single factor model to evaluate assets. Consider the following data for portfolios A, B, and C:

<u>Portfolio</u>	<u>Expected Return</u>	<u>Beta</u>
A	10%	1.0
B	20%	2.0
C	13%	1.5

Calculate the arbitrage opportunity from the data provided. *Hint: Create a portfolio with 50% in A and 50% in B, and then compare the expected return and beta of that portfolio to Portfolio C.*



Professor's Note: A portfolio beta equals the weighted average of the individual asset betas, and likewise, the portfolio expected return equals the weighted average of the individual asset expected returns.

Answer:

By allocating 50% to portfolios A and B, we can obtain a portfolio (D) with beta equal to the Portfolio C beta (1.50):

$$\text{beta for Portfolio D} = 0.50(1) + 0.50(2) = 1.50$$

While the betas for portfolios D and C are identical, the expected returns are different:

$$\text{expected return for Portfolio D} = 0.50(0.10) + 0.50(0.20) = 0.15 = 15\%$$

Therefore, we have created Portfolio D that has the same risk as Portfolio C (beta = 1.50) but has a higher expected return than Portfolio C (15% versus 13%). By purchasing Portfolio D and short-selling Portfolio C, we expect to earn a 2% return (15% minus 13%).

The portfolio that is long Portfolio D and short Portfolio C is called the **arbitrage portfolio**. We have invested nothing up front because we merely use the proceeds of the short sale on Portfolio C to purchase Portfolio D. And, we have undertaken no systematic risk. The beta on our investment equals the difference in betas between our long and short positions: $1.5 - 1.5 = 0$. As investors exploit the arbitrage opportunity, prices of assets in Portfolio C will drop, and the (future) expected return for Portfolio C will rise to its equilibrium value.

The APT assumes there are no market imperfections preventing investors from exploiting arbitrage opportunities. As a result, extreme long and short positions are permitted, and mispricing will disappear immediately. Therefore, all arbitrage opportunities such as the one described in the previous example would be exploited and eliminated immediately.

THE RELATIONSHIP BETWEEN CAPM AND APT

AIM 5.3: Explain the relationship between the CAPM and the APT.

Both the arbitrage pricing model and the capital asset pricing model describe equilibrium expected returns for assets. Moreover, the CAPM is a special case of APT with only one factor exposure—market risk.

If stock returns follow a single factor model with that factor being market risk, then Equation 1 would be:

$$R_n = \alpha + \beta_n \times R_M + u_n$$

Equation 2 would then be:

$$E(R_n) = R_F + [E(R_M) - R_F]\beta_n$$

As you can see, this equation is the capital asset pricing model.

APPLYING THE APT TO ACTIVE AND PASSIVE INVESTMENT MANAGEMENT

AIM 5.4: Describe how the APT can be used in both active and passive portfolio management.

Passive portfolio management attempts to track an index as closely as possible, while active management attempts to beat a benchmark by making bets that are more speculative in nature. The goal of active management is to earn a positive active return, also called alpha. The APT can be applied in both cases.

In a *passive management* framework, a manager can replicate an index by holding all securities in that index but doing so comes at the cost of transacting in all of the securities. Another approach is to replicate the returns of the index while holding a smaller subset of securities. A single index model could be used to create a portfolio with beta equal to one, but this approach does not control for other risk factors that may have an impact on security returns. Using a multifactor model based on APT allows a manager to control a number of factors that affect security returns so that the passive portfolio has risk exposures equivalent to its specified index.

Beyond matching an index, a passive manager can apply APT in other situations where the single index model or replication are not appropriate.

- Some pensions and endowments may take part or implement a form of socially responsible investing, whereby the fund chooses not to invest in companies affiliated with (among other things) gambling and tobacco. Using an APT derived model, it is possible to construct portfolios that exclude these stocks but matches the major risk factors of the target index. Using a single factor model would not ensure that the portfolio matches all risk factors.
- A pension fund or endowment may wish to match the risk exposures for a liability (or quasi-liability) that the portfolio is funding. For example, a firm's pension payments may be tied to inflation. Matching a portfolio to an index by using replication or the single index model would not account for this movement, but an APT derived multifactor model that includes a risk factor for inflation would likely be able to satisfy this risk exposure match.
- Some investors may have cash flows tied to factors that are not priced risk factors, but by including these additional factors into the APT framework, an investor can align the portfolio's sensitivities to his cash flows.

In an *active management* framework, the APT allows investors to make active bets regarding factors. An investor benchmarked to the Russell 2000 may believe economic growth to be higher than forecasted. It would be possible using the APT to increase the portfolio's exposure to economic growth beyond the exposure of the Russell 2000, making a factor bet on economic growth.

Additionally, the APT can be applied in active management as an equilibrium-pricing model to determine the required return for a security. A fund may employ an analyst to determine whether securities are over or undervalued. To determine whether securities are fairly valued or not, the analyst needs to know the required return for the security. One possible approach is to use the CAPM; however, this model ignores all possible risk factors other than market risk. If a multifactor model is correct, and CAPM is used, then securities with different risk exposures, but equivalent market exposures, will be incorrectly identified as having identical required rates of return.

The primary use of the APT is to construct a portfolio that closely tracks a benchmark while earning a higher return. There are several methods that can be applied to accomplish this objective. For example, a portfolio could be constructed using APT to match the returns of the benchmark (similar to the passive management discussion) but only use stocks that are ranked as “buys” by a team of analysts. Alternatively, quantitative methods could be used to rank stocks or forecast the return of stocks.

KEY CONCEPTS

1. The arbitrage pricing theory (APT) model assumes:
 - Returns are derived from a multifactor model.
 - Unsystematic risk can be completely diversified away.
 - No arbitrage opportunities exist.

APT assumes that returns follow the form:

$$R_n = \alpha_n + \beta_{n,1} \times F_1 + \dots + \beta_{n,k} \times F_k + u_n$$

where:

- R_n = return for stock n
- α_n = expected return when factor exposures are all zero
- $\beta_{n,k}$ = k th factor exposure for stock n
- F_k = factor k
- u_n = idiosyncratic return for stock n

If returns follow this form, then the expected return for a security is given as:

$$E(R_n) = \lambda_0 + \lambda_1 \beta_{n,1} + \dots + \lambda_k \beta_{n,k}$$

where:

- λ_0 = R_F
- λ_k = expected risk premium associated with each risk factor

2. Given factor exposures, the risk-free rate, and expected factor risk premiums, it is straightforward to calculate the expected return on a security using APT.
3. The CAPM is a special case of APT where there is only one priced risk factor (market risk).
4. Passive portfolio management attempts to track an index as closely as possible, while active management attempts to beat a benchmark by making bets.

APT allows passive managers to match multiple risk exposures of an index using a smaller set of securities. This allows managers to:

- Engage in socially responsible investing while matching an index's risk exposures.
- Match the risk exposures for a liability (or quasi-liability).
- Match portfolio sensitivities to cash flows.

In an active management framework, the APT can be applied as follows:

- Engage in factor bets by altering factor exposure relative to a benchmark.
- Determine whether stocks are fairly valued by using APT to generate the required rate of return.
- Construct portfolios that match benchmark risk exposures but earn active returns by using analyst recommendations, a quantitative ranking, or forecasted returns.

CONCEPT CHECKERS

1. Which of the following statements is most likely an assumption of the APT?
Expected returns:
 - A. are linearly related to market risk exposure.
 - B. are related to systematic and unsystematic risk.
 - C. for securities with identical risk exposures will be identical.
 - D. are linearly related to multiple risk factors that are clearly defined.
2. Using an APT model, what is the expected return for a stock given the following factor exposures and returns? Assume the risk-free rate is equal to 3%.

Factor exposures:
 - Standardized probability of default: 0.5.
 - Standardized average daily trading volume: -0.2.
 - Standardized average earnings growth forecast: 1.5.
Expected factor risk premiums:
 - Standardized probability of default: 2%.
 - Standardized average daily trading volume: -1%.
 - Standardized average earnings growth forecast: 1.5%.
 - A. 4.8%.
 - B. 6.1%.
 - C. 6.5%.
 - D. 7.5%.
3. Which of the following statements regarding the CAPM and APT is most likely correct?
 - A. CAPM and APT are unrelated.
 - B. CAPM is a special case of APT with only one factor.
 - C. APT is a special case of CAPM with multiple factors.
 - D. CAPM is a special case of APT with multiple factors.
4. A passive portfolio manager would be least likely to use APT to construct a portfolio that:
 - A. tracks an index using a small number of securities.
 - B. matches the risk exposures of a benchmark but earns larger returns.
 - C. matches the risk exposures as an index but excludes tobacco and gambling stocks.
 - D. matches most of the risk exposures of an index and alters the factor exposure to inflation to align the exposure to portfolio liabilities.

5. At a recent analyst meeting at Invest Forum, analysts Michelle White and Ted Jones discussed the use of the capital asset pricing model (CAPM) and arbitrage pricing theory (APT). White states that the CAPM implies that investors hold a risky portfolio of U.S. assets weighted according to their relative market value capitalizations. Jones states that the APT implies that investors will make investment decisions by allocating their money between a risk-free asset and the market portfolio. Are the statements of White and Jones correct?
- A. Both statements are correct.
 - B. Only Jones's statement is correct.
 - C. Only White's statement is correct.
 - D. Neither statement is correct.

CONCEPT CHECKER ANSWERS

1. C APT assumes that the process of arbitrage will ensure that securities with identical risk exposures will have the same expected return. APT assumes that returns are linearly related to multiple risk factors, but the theory does not specify what the risk factors are. Unsystematic risk (stock specific risk) can be completely diversified away leaving only systematic risk factors. Expected returns are only related to these systematic risk factors.
2. C Given the factor exposures and returns, the expected return is calculated as follows:
$$E(R_n) = 3\% + 0.5 \times 2\% + (-0.2) \times (-1\%) + 1.5 \times 1.5\%$$
$$E(R_n) = 6.5\%$$
3. B The CAPM is a special case of APT with only one factor—the market risk premium.
4. B Passive portfolio management attempts to track an index as closely as possible, while active management attempts to earn larger returns than a benchmark by making active bets. The other answer choices all describe ways that APT can be used to implement passive portfolio management.
5. D The CAPM assumes all investors have identical expectations and use mean-variance analysis, implying that they all identify the same risky tangency portfolio (the “market portfolio”) and combine that risky portfolio with the risk-free asset when creating their portfolios. Because all investors hold the same risky portfolio, the weight on each asset must be equal to the proportion of its market value to the market value of the entire portfolio. White is incorrect, however, because the market portfolio would include more than just U.S. assets. In contrast, the APT places no special emphasis on the market portfolio. In fact, the APT does not even require that the “market portfolio” exist. Therefore, Jones is incorrect.

APPLYING THE CAPM TO PERFORMANCE MEASUREMENT: SINGLE-INDEX PERFORMANCE MEASUREMENT INDICATORS

Topic 6

EXAM FOCUS

This topic further expands on the concepts of the capital market line and the security market line by examining measures used to assess portfolio performance on a risk-adjusted basis. In a previous topic, we mentioned that the risk-to-reward ratio for the capital market line (i.e., its slope) is known as the Sharpe ratio. In addition, we discussed how to assess a portfolio's alpha return when comparing actual performance to expected performance based on the CAPM. The formal expression for this calculation is known as Jensen's alpha. The Treynor measure is another popular performance metric, and is similar to the Sharpe ratio but uses beta as the risk measure instead of standard deviation. Toward the end of this topic, we examine additional risk-return assessment measures such as the information ratio and the Sortino ratio. In general, all of the performance measures introduced evaluate excess return over some form of risk. For the exam, memorize these measures of performance since they are popular concepts to test.

MEASURES OF PERFORMANCE

AIM 6.1: Calculate, compare, and evaluate the Treynor measure, the Sharpe measure, and Jensen's alpha.

Modern portfolio theory and the CAPM are built upon the link between risk and return. Three measures exist to assess an asset's or portfolio's return with respect to its risk.

- The **Treynor measure** is equal to the risk premium divided by beta, or systematic risk:

$$\text{Treynor measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\beta_P} \right]$$

- The **Sharpe measure** is equal to the risk premium divided by the standard deviation, or total risk:

$$\text{Sharpe measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\sigma_P} \right]$$

- The **Jensen measure** (or Jensen's alpha or just **alpha**), is the asset's excess return over the return predicted by the CAPM:

$$\text{Jensen measure of a portfolio} = \alpha_p = E(R_p) - [R_F + [E(R_M) - R_F]\beta_p]$$

In all three cases, for a given portfolio, the higher, the better. The two that are most similar are the Treynor and Sharpe measures. They both normalize the risk premium by dividing by a measure of risk. Investors can apply the Sharpe measure to all portfolios because it uses total risk, and it is more widely used than the other two measures. The Treynor measure is more appropriate for comparing well-diversified portfolios. Jensen's alpha is the most appropriate for comparing portfolios that have the same beta.

Some consider the Sharpe measure a better method for measuring historical performance. Since betas must be estimated and the portfolio beta is the average of the betas of assets in a portfolio, the Treynor measure is considered a more forward-looking measure.

In addition to these comparisons, it is useful to realize that some relationships exist between the measures. For instance:

$$\text{Treynor measure} = \frac{\alpha_p}{\beta_p} + [E(R_M) - R_F]$$

For a well-diversified portfolio we can use the following approximation: $\beta_p \approx \frac{\sigma_p}{\sigma_M}$.

Substituting this into the expression for Jensen's alpha and applying some algebra gives us:

$$\text{Sharpe measure} \approx \frac{\alpha_p}{\sigma_p} + \frac{E(R_M) - R_F}{\sigma_M}$$

Applying the approximation $\beta_p \approx \frac{\sigma_p}{\sigma_M}$ again gives us:

$$\text{Sharpe measure} \approx \left[\frac{\text{Treynor measure}}{\sigma_M} \right]$$



Professor's Note: Do not focus too much attention on these approximations. The key to this AIM is understanding how to calculate the three measures of performance as is shown in the following example.

Example: Calculating performance measures

For a portfolio of ten stocks, we may find, via fundamental analysis estimates of the individual stocks, that the portfolio's expected return is 14% with a standard deviation of 25%. The beta of the portfolio is 1.1. The expected return of the market is 12.5% with a standard deviation of 20.2%. The risk-free rate is 2.6%. Calculate the Treynor, Sharpe, and Jensen measures.

Answer:

$$\text{Treynor measure} = \left[\frac{E(R_P) - R_F}{\beta_P} \right] = \left[\frac{0.14 - 0.026}{1.1} \right] = 0.1036$$

$$\text{Sharpe measure} = \left[\frac{E(R_P) - R_F}{\sigma_P} \right] = \left[\frac{0.14 - 0.026}{0.25} \right] = 0.456$$

$$\begin{aligned} \text{Jensen measure} &= \alpha_P = E(R_P) - [R_F + [E(R_M) - R_F]\beta_P] \\ &= 0.14 - [0.026 + (0.125 - 0.026)(1.1)] = 0.0051 \end{aligned}$$

We can compare these to the corresponding measures of the market portfolio:

$$\text{Treynor measure of the market} = \left[\frac{0.125 - 0.026}{1} \right] = 0.099$$

$$\text{Sharpe measure of the market} = \left[\frac{0.125 - 0.026}{0.202} \right] = 0.4901$$

$$\text{Jensen measure of a portfolio} = 0.125 - [0.026 + (0.125 - 0.026)(1)] = 0.0$$

Based upon the Treynor measure and the Jensen measure of the preceding example, the portfolio of ten stocks is superior to the market. However, the relationship is reversed using the Sharpe measure. This implies that the manager has selected ten stocks that offer superior returns relative to their systematic risk; however, a 10-stock portfolio is much less diversified than the market. The standard deviation for the 10-stock portfolio ($\sigma_P = 25\%$), when compared to $\sigma_M = 20.2\%$, reflects the lower level of diversification.

Extensions to Jensen's Alpha

There are several ways to modify or extend the Jensen measure. Since Jensen's measure is simply a raw return in excess of some reference (i.e., that implied by the CAPM in the case of the standard Jensen's measure) we can simply replace that reference with a value that we feel is more appropriate. One reference would be the required return based on the CML. The manager has created a portfolio with risk σ_P , which then has a reference return equal to $E(R_{\text{reference}})$ as given by the equation:

$$E(R_{\text{reference}}) = R_F + [E(R_M) - R_F] \left[\frac{\sigma_P}{\sigma_M} \right]$$

The alpha in this case would be the portfolio's return minus the reference return:

$$\alpha = E(R_p) - E(R_{\text{reference}})$$

Other extensions of Jensen's measure would use a measure of $E(R_{\text{reference}})$ derived from a multifactor model (i.e., more than one independent variable). Another value of $E(R_{\text{reference}})$ could be derived from a variation of the CAPM called the Black model, which uses the return on a "zero-beta" portfolio in place of the risk-free rate. In all cases, the idea is the same: measure the raw return difference of the managed portfolio against the required return given its level of risk.

AIM 6.2: Compute and interpret tracking error, the information ratio, and the Sortino ratio.

If a manager is trying to earn a return higher than the market portfolio or any other reference or benchmark, the difference will have some variability over time. In other words, even if the manager is successful in generating a positive alpha, the alpha will vary over time. **Tracking error** is the term used to describe the standard deviation of the difference between the portfolio return and the benchmark return. This source of variability is another source of risk to use in assessing the manager's success. Typically, the manager must keep the tracking error below a stated threshold. The manager must weigh transactions and other costs in managing the portfolio to reduce tracking error against the extra risk it introduces into the management process.

The **information ratio** is essentially the alpha of the managed portfolio relative to its benchmark divided by the tracking error. If we let R_B denote the return of the benchmark we can write:

$$e_p = R_p - R_B$$

$$\text{tracking error} = \sigma_{e_p}$$

$$\text{information ratio} = \left[\frac{E(R_p) - E(R_B)}{\sigma_{e_p}} \right] = \frac{\alpha_p}{\sigma_{e_p}}$$

This is a measure used to assess if the manager's deviation from the benchmark has reaped an appropriate return. It is called an "information ratio" because it is essentially a measure of how well the manager has acquired and used information compared to the average manager.

Example: Calculating the information ratio

A manager typically generates an alpha of 1.5% with a tracking error of 2.25%. Calculate the information ratio.

Answer:

$$\text{information ratio} = \left[\frac{1.5}{2.25} \right] = 0.667$$

The **Sortino ratio** is similar to the Sharpe ratio except for two changes. We replace the risk-free rate with a minimum acceptable return, denoted R_{\min} , and we replace the standard deviation with a type of semi-standard deviation. A semi-standard deviation measures the variability of only those returns that fall below the minimum acceptable return. The measure of risk in the Sortino ratio is the square root of the mean squared deviation from R_{\min} of those observations in time periods t where $R_{Pt} < R_{\min}$, else zero. Letting R_{\min} denote the minimal acceptable return and MSD_{\min} the risk measure:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{\min}}{\sqrt{MSD_{\min}}}$$

where:

$$MSD_{\min} = \frac{\sum_{R_{Pt} < R_{\min}} (R_{Pt} - R_{\min})^2}{N}$$

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

Example: Calculating tracking error, the information ratio, and the Sortino ratio

Over a 10-year period, a manager uses a covered call strategy to enhance the return of the index fund she manages. The record of the fund's returns is (0.095, 0.08, -0.022, 0.11, 0.09, -0.05, -0.035, 0.124, 0.072, 0.055). The corresponding benchmark returns record is (0.087, 0.078, -0.034, 0.124, 0.10, -0.064, -0.042, 0.131, 0.062, 0.059). The minimum acceptable return is 4%. Calculate the tracking error, the information ratio, and the Sortino ratio. Assume mean squared deviation (min) = 0.0017569.

Answer:

To compute the tracking error, the first step is to compute the differences between the portfolio and the benchmark. Those differences are: (0.008, 0.002, 0.012, -0.014, -0.01, 0.014, 0.007, -0.007, 0.01, -0.004). The tracking error is the standard deviation of these numbers:

$$\text{mean} = \alpha = 0.0018$$

$$\text{tracking error} = 0.00992$$

To compute the information ratio, divide the mean by the tracking error:

$$\text{information ratio} = 0.0018 / 0.00992 = 0.1815$$

The Sortino ratio is the mean of the ten portfolio returns minus 4%, which is $0.0519 - 0.04 = 0.0119$, divided by the square root of MSD_{\min} .

$$MSD_{\min} = 0.0017569$$

$$\text{Sortino ratio} = 0.0119 / 0.0419 = 0.2840$$



Professor's Note: In Book 2, we will examine the formula for standard deviation that is used in the tracking error calculation above. It is based on the sum of the squared differences between each data point and the mean. This sum is then divided by the number of observations adjusted for degrees of freedom (in this case $n - 1$). The square root of the computed value will be the standard deviation.

RECENT RISK-ADJUSTED PERFORMANCE MEASURES

AIM 6.3: Explain the Morningstar Rating System, VaR based, and management related risk-adjusted return measures.

Recent investment studies have produced variations on traditional risk-adjusted performance measures. Here we explain Morningstar rankings, VaR based risk-adjusted measures, and performance measures that account for investment style.

Morningstar Rating System

The firm Morningstar evaluates investment funds by ranking risk-adjusted performance within a specified peer group and then assigning a star rating to each fund within the group. Those funds in the top 10% of their peer group will obtain a five star rating, while funds in the bottom 10% will obtain a one star rating. The middle 35% will be assigned a three star rating, while those funds above the middle 35% but below the top 10% will earn four stars, and those funds below the middle 35% but above the bottom 10% will earn two stars.

The current Morningstar rating system classifies equity and debt funds based on 48 different peer groups. Equity funds, for example, are grouped based on both strategy (growth, value, or blend) and size (large-cap, mid-cap, small-cap). Since funds are only evaluated against the risk-adjusted performance of their peer group, Morningstar is able to identify the best funds, unbiased of how each peer group is performing relative to the broader market.

Within each peer group, funds are assigned a star rating based on their risk-adjusted ranking (RAR). A fund's RAR is computed as:

$$RAR_i = \text{relative return}_i - \text{relative risk}_i$$

Relative return is computed as fund return divided by the average return of the peer group. If the average return is less than the risk-free rate, then relative return should instead divide fund return by the risk-free rate. **Relative risk** is obtained by dividing a fund's risk level by the average risk of funds in the peer group. Fund risk is measured based on not only downside volatility, but also upside volatility. By analyzing volatility to both the upside and the downside, the RAR measure is not skewed by high-risk funds with strong short-term performance. As a result, it is difficult for high-risk funds to earn high star ratings.

Once each fund's RAR is calculated, the funds are ranked from best to worst within each peer group, and a star rating is subsequently assigned. The time horizon for computing the RAR is one month. This, however, presents a limitation of the Morningstar rating system since most funds have a longer time horizon.

VaR Based Risk-Adjusted Measures

Value at risk (VaR) identifies the maximum loss that a fund can sustain given a stated level of confidence. VaR offers a unique perspective on risk compared to standard deviation or beta, so it can be useful to include VaR as a risk-adjusted performance indicator. By modifying the Sharpe ratio, we can analyze risk-return performance based on value at risk (as shown in the following equation). Note that when comparing funds, the portfolio VaRs must use the same level of confidence.

$$\frac{R_p - R_F}{\text{VaR}_p / V_p}$$

where:

R_p = portfolio return

R_F = risk-free rate

VaR_p = portfolio VaR

V_p = initial portfolio value

VaR can also be used during the investment decision-making process. For example, if a portfolio manager wants to add a security to a fund, the portfolio VaR before the addition can be compared to the portfolio VaR after the addition to check which potential investments would be favorable in terms of the risk-return tradeoff. The increase in portfolio VaR from the addition of an investment is known as **incremental VaR (IVaR)**. The fund manager would be interested in adding assets that have a low IVaR relative to the incremental expected return.

Management Related Risk-Adjusted Measures

Traditional risk-adjusted performance measures are useful for evaluating fund performance in comparison with the broader market (i.e., the market portfolio). One limitation, however, is that these measures do not account for investment style. A simple remedy is to use a modified capital market line equation to compare risk-adjusted return for a fund with the risk-adjusted return for the benchmark that appropriately captures the investment style of the fund. This measure is known as **style risk-adjusted performance (SRAP)** and involves

finding the difference between the RAP of the fund and the RAP of the fund's benchmark.

Risk-adjusted performance for an asset is calculated as follows:

$$RAP_p = R_F + \frac{\sigma_M}{\sigma_p} \times (R_p - R_F)$$

where:

R_F = risk-free rate

σ_M = market return standard deviation

σ_p = portfolio return standard deviation

R_p = portfolio return

An important part of computing the relative performance of a fund is finding the correct benchmark/index to use. To evaluate investment style, one would calculate the difference between the fund RAP and the market RAP and then compare that figure to the difference between the fund RAP and the benchmark RAP (i.e., the SRAP). In most cases, adjusting for management style will produce an SRAP that differs from the relative RAP of the fund compared to the market.

Risk-adjusted performance indicators have also been developed to compare managers within a peer group. The **correlation-adjusted performance measure** improves on traditional performance measures, such as the Sharpe ratio, by including both standard deviation and correlation between funds. The main application of this type of performance indicator is to allow an investor to find the optimal allocation across fund managers that maximizes desired return while minimizing risk.

KEY CONCEPTS

1. Three commonly used risk/return measures are:
 - Treynor measure of a portfolio = $\left[\frac{E(R_P) - R_F}{\beta_P} \right]$
 - Sharpe measure of a portfolio = $\left[\frac{E(R_P) - R_F}{\sigma_P} \right]$
 - Jensen measure of a portfolio = $\alpha_P = E(R_P) - [R_F + [E(R_M) - R_F]\beta_P]$
2. The three risk measures above give different perspectives and may give different rankings for portfolios. A portfolio with low diversification may have a higher Treynor measure, a higher alpha, but a lower Sharpe measure than another portfolio.
3. Alpha can be modified by the use of other reference portfolios.
4. Tracking error and the information ratio build upon Jensen's alpha. Tracking error is the standard deviation of alpha over time. The information ratio is the average alpha over time divided by the tracking error.
5. The Sortino ratio should be used when there is more focus on the likelihood of loss:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{\min}}{\sqrt{\text{MSD}_{\min}}}$$

The MSD_{\min} is a semi-variance that only measures the variability of the portfolio's return observations below R_{\min} .

6. Recent investment studies have produced variations on traditional risk-adjusted performance measures. New methods include Morningstar ratings, VaR based measures, and measures that account for investment style.

CONCEPT CHECKERS

1. For a given portfolio, having a Treynor measure greater than the market but a Sharpe measure that is less than the market would most likely indicate that the portfolio is:
 - A. not well diversified.
 - B. generating a negative alpha.
 - C. borrowing at the risk-free rate.
 - D. not borrowing at the risk-free rate.

2. With respect to performance measures, the use of the standard deviation of portfolio returns is a distinguishing feature of the:
 - A. beta measure.
 - B. Jensen measure.
 - C. Sharpe measure.
 - D. Treynor measure.

3. For a given portfolio, the expected return is 9% with a standard deviation of 16%. The beta of the portfolio is 0.8. The expected return of the market is 12% with a standard deviation of 20%. The risk-free rate is 3%. The portfolio's alpha is:
 - A. -1.2%.
 - B. -0.6%.
 - C. +0.6%.
 - D. +1.2%.

4. You are given the following information:

Risk-free rate	4%
Minimum acceptable return	6%
Benchmark return	10%
Expected return on portfolio	12%
Expected return on market	9%
Beta	1.25
Standard deviation (portfolio)	7.3%
Semi-standard deviation (portfolio)	8.2%

The Sortino ratio of the portfolio is closest to:

 - A. 0.24.
 - B. 0.73.
 - C. 0.82.
 - D. 0.98.

5. An analyst has compiled the following data on Stock P:

Covariance _{P, market}	0.0315
$\sigma_{\text{Stock P}}$	16.50%
σ_{market}	15.00%
Expected market return	11.80%
Risk-free rate	4.50%
Stock P actual return	13.25%

Calculate and interpret Jensen's Alpha for Stock P.

- A. +1.47% overperformed.
- B. -1.47% underperformed.
- C. +1.45% overperformed.
- D. -1.45% underperformed.

CONCEPT CHECKER ANSWERS

1. **A** Low diversification can produce this result because it will likely increase the standard deviation of the portfolio's returns, thus decreasing its Sharpe ratio. Using margin is not directly related to the risk-adjusted performance because adjusting for risk removes the effect of leverage. A Treynor ratio greater than the market Treynor ratio would result in a positive alpha (not a negative alpha).
2. **C** The Sharpe measure is the portfolio return minus the risk-free rate divided by the standard deviation of the return. The Treynor and Jensen measures use beta. The answer "beta measure" is a nonsensical choice for this question.
3. **A** The alpha is $9\% - [3\% + 0.8 \times (12\% - 3\%)] = -1.2\%$.
4. **B** $(\text{portfolio return} - \text{minimum acceptable return}) / \text{semi-standard deviation}$

$$(0.12 - 0.06) / 0.082 = 0.7317$$

Choice A is incorrect because it uses the benchmark return in the numerator instead of the minimum acceptable return.

Choice C is incorrect because it uses the standard deviation in the denominator instead of the semi-standard deviation.

Choice D is incorrect because it uses the risk-free rate in the numerator instead of the minimum acceptable return.

5. **B** Jensen's Alpha = actual return – CAPM expected return

$$\text{CAPM: } E(R) = R_F + \beta(R_M - R_F)$$

$$\beta = \frac{\text{covariance}_{P, \text{market}}}{\text{variance}_{\text{market}}}$$

Step 1: Calculate β

$$\beta = 0.0315 / 0.15^2 \quad \beta = 1.4$$

Step 2: Calculate the CAPM expected return

$$E(R) = 4.5 + 1.4(11.80 - 4.5) = 14.72\%$$

Step 3: Calculate Jensen's Alpha

$$\text{Jensen's Alpha} = \text{actual return} - \text{CAPM } E(R) = 13.25\% - 14.72\% = -1.47\%$$

Stock P has underperformed the market by 1.47% when taking into account its level of systematic risk as measured by beta.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

OVERVIEW OF ENTERPRISE RISK MANAGEMENT

Topic 7

EXAM FOCUS

This is an important topic on enterprise risk management (ERM). Although the assigned reading was published almost eight years ago (before major events affecting risk management, such as the credit crisis of 2007 to 2009), the basic principles still apply to risk management in the current decade. This topic is primarily qualitative and extremely focused so all of the concepts covered here are important for exam purposes. Some of the discussion regarding quantitative and statistical concepts will tie in with other parts of the curriculum. As a result, you may benefit from studying this article again after you get through the entire Part I curriculum. Note that some of these concepts also overlap with the integrated risk management material in the Part II curriculum.

DEFINITION OF ERM

AIM 7.1: Describe what is meant by ERM.

ERM takes an integrated, big-picture approach to the risk management process, which is a departure from separately managing individual risks within an organization. Decisions are made on an overall basis and therefore, it positively impacts decision-making throughout the organization. Additionally, ERM considers the concept of risk management as having an opportunistic side.

The Casualty Actuarial Society defines ERM as:

“...the discipline by which an organization in any industry assesses, controls, exploits, finances, and monitors risks from all sources for the purpose of increasing the organization’s short- and long-term value to its stakeholders.”

We should analyze the wording carefully to fully appreciate the definition.

- *Discipline*—a formal process for the organization that is followed by the management team and is an integral part of the organization’s culture.
- *Any industry*—the definition applies to industries beyond merely insurance.
- *Exploits...increasing the organization’s short-and long term value*—ERM has a dual focus: to reduce risk and to seek business opportunities.
- *Stakeholders*—includes shareholders, debtholders, management, employees, and customers.

RISKS ADDRESSED BY ERM

AIM 7.2: Identify and describe risks addressed by ERM.

There are four types of risks covered in the ERM framework: (1) hazard risks, (2) financial risks, (3) operational risks, and (4) strategic risks. Not all risks can be exactly classified into one of these categories, but more noteworthy is the fact that ERM is comprehensive in its coverage of major risk factors impacting an organization. General examples of each of the four types are used as illustrations.

Hazard Risks

To enhance understanding of hazard risks, they can be further broken down into three subcategories.

1. First-party hazard risks: examples include fire and weather causing property destruction, business interruption, and theft.
2. Second-party hazard risks: injuries and illnesses related to employees (personal or work-related).
3. Third-party hazard risks: liability relating to injuries to visitors on premises, product recalls, and defective products.

Financial Risks

- Price risk—interest rate, price, foreign exchange, commodity (price of input).
- Liquidity risk—cash flow crunch; inability to buy/sell item at a reasonable amount.
- Credit risk—default or downgrade in rating.
- Inflation/purchasing power risk—high inflation rate reduces purchasing power.
- Hedging/basis risk—price movements between hedging instrument and item being hedged do not move to the same degree.

Operational Risks

- General business operations—risk of inefficiency and ineffectiveness.
- Empowerment—risk of incompetent leadership/management at the top and throughout the organizational ranks.
- Information technology—risk of obsolescence and inability of information technology to function in a changing operational environment.
- Information/business reporting—risk of lack of reliability (i.e., poor controls, untrained staff) and lack of relevance of accounting information used for decision-making purposes.

Strategic Risks

- Reputational damage—risk of losses resulting from substandard product/service, lawsuit, and so forth.
- Competition—risk of competitors developing a better product/service (e.g., iPhone versus Blackberry).

- Changing societal trends—concern with health and fitness (e.g., Krispy Kreme doughnuts on the decline).
- Technological innovation—online versus print format (e.g., newspapers on the decline).
- Regulatory and political trends—stricter vehicle emissions laws (e.g., larger automobiles on the decline).

MEASURES, MODELS, AND TOOLS USED IN AN ERM FRAMEWORK

AIM 7.3: Describe the measures, models, and tools typically used within an ERM framework.

Measures

Solvency-related measures: Focus on the bottom end of the probability distribution—measure level of capital required to handle negative, as well as worst-case scenarios.

- Shortfall risk—the probability that an amount falls below a specific threshold level.
- Probability of ruin—related to shortfall risk; the percentile at which point (threshold level) capital is completely used up; the lowest acceptable probability of ruin is stated and from there, an organization can determine its economic capital.
- Value at risk (VaR) —the maximum loss to occur at a given level of probability, given a specific period of time and normal market conditions.
- Expected policyholder deficit (EPD) or economic cost of ruin (ECOR)—extension of probability of ruin as it also considers the severity of the ruin (the expected value of the shortfall).
- Tail events—unlikely and extreme events with big losses.
- Tail Value at Risk (Tail VaR) or Tail Conditional Expectation (TCE)—similar to ECOR because it considers the probability and severity of ruin; however, it considers the severity of all events beyond the tail threshold amount (which is larger than the shortfall amount).

Performance-related measures: Focus on the middle of the probability distribution—measure the volatility of results on an on-going basis.

- Variance—the average squared difference between a random variable and its mean; considers the amount of variation of the values, taking into account all possible values and their probabilities.
- Standard deviation—the square root of variance.
- Semi-variance and downside standard deviation—similar to standard deviation, but rather than all possible values, only negative deviations from a stated target level are examined.
- Below-target-risk (BTR)—expected value of negative deviations of a random variable from a stated target level.

Models

Risk modeling is the process by which solvency and performance-based risk measures are developed.

Structural (causal) financial models clearly indicate the cause/effect relationships. As a general example, inputs such as risk factors (e.g., interest rates) and corporate strategies will determine outcomes, such as revenue and earnings growth. Such structural financial models are usually deterministic models because the expected outcomes calculated from the inputs do not consider the probabilities of outcomes different from the expected outcomes.

Deterministic models could be transformed into stochastic (probabilistic) models by assuming certain inputs to be variable. The inputs of the models (and hence the outcomes) are assumed to be uncertain, and so a probability distribution around the expected value of a given variable becomes the input to a stochastic model.

Two primary types of stochastic modeling approaches are as follows:

1. *Statistical analytic models*—Such models consider observations only and do not consider cause/effect relationships. They are easy to parameterize; for example, means, variances, and correlations are examined. The calculation technique involves analytic methods (closed form solutions derived by solving a series of equations with numerous simplifying assumptions). Given the simplicity, the models are appropriate for analyzing publicly traded companies at minimal cost.
2. *Structural simulation models (Monte Carlo)*—Such models are based on clear cause/effect relationships. Many iterations are often required, but the models can handle complex situations and can mimic almost any situation contemplated (very flexible). Given the complexity, the models are generally appropriate for analyzing private companies.

Tools

Decision-making applications of the preceding risk management concepts are categorized as generic and specific.

Generic Applications

- *Optimization*—Consists of a mathematical process whereby a number of options are presented along with constraints, a probability distribution of uncertain conditions, and a particular item to be maximized (i.e., return) or minimized (i.e., risk). A good example would be the efficient frontier in a risk/return tradeoff analysis—the portfolio combinations lying on the efficient frontier are *equally good*, but the choice depends on risk/return preferences.
- *Candidate analysis*—A specific example of optimization whereby only a certain number of options are considered and the best of those are selected after the analysis. The options are compared to one another on an absolute basis in terms of risk/return.

Specific Applications

- *Capital management*—four specific applications in this category.
 - ♦ Capital adequacy: What is the minimum amount of capital required to meet a certain economic capital constraint?
 - ♦ Capital structure: What is the optimal mix of capital given the risks and financial objectives of the organization?
 - ♦ Capital attribution: How to assign capital to the different business segments based on their respective relative risks? Must also consider the diversification credit—suggests that total economic capital required for the organization is less than the total of the individual capital requirements of the business segments.
 - ♦ Capital allocation: How much capital to actually distribute to the various segments of the organization?
- *Performance measurement*—risk-based measures (i.e., Treynor, Sharpe, Jensen measures) to evaluate performance of individual business segments.
- *Investment strategy/asset allocation*—determine optimal mix of assets (by asset class) to maximize return for a given level of risk. Can extend the analysis to include liabilities, too.
- *Insurance/reinsurance/hedging strategy optimization*—this optimization is often done through candidate analysis and takes into account program costs and risk reduction capabilities (i.e., reduce required economic capital or cost of capital).
- *Crisis management*—planning ahead to be able to handle a material and negative event that would otherwise impair an organization's ability to function.
- *Contingency planning*—similar to crisis management but involves the actual process of devising and implementing the procedures to be followed in case of a crisis.
- *Business expansion/contraction strategy*—analyzing potential mergers, acquisitions, and divestitures in context of their marginal impact on an organization's level of risk.
- *Distribution channel strategy*—using simulation analysis to evaluate alternative distribution channels and their financial impacts (i.e., market share, profits) on a risk/return basis.
- *Strategic planning*—using structural simulation modeling to evaluate and select among alternative business strategies.

ERM Implementation

AIM 7.4: Discuss practical considerations related to ERM implementation.

There are at least eight general points to consider in ensuring a proper ERM implementation.

1. *Appointing an appropriate ERM leader.*

Designating a qualified individual to assume the Chief Risk Officer (CRO) role is the first step in implementing ERM. It is necessary for the CRO to have a complete understanding of all the organization's risks and to have the power to actually make necessary changes in business operations to manage risks.

2. *Integrating ERM into the organization's culture.*

The past process of risk management in an organization was fragmented and often managed by department (i.e., human resources, treasury, marketing, information technology). Each department would likely use different approaches to risk management. A more modern view of risk management would entail the use of ERM to work together with the various departments to result in some integration across the organization while also maintaining a reasonable level of flexibility within each department.

3. *Researching and gathering all potential risks in the organization.*

There are many risks to an organization that are difficult to identify or that are not even considered given their obscurity. Yet through time, they have occurred and demonstrated themselves to be large in scope (i.e., environmental and terrorism threats). Therefore, it is necessary to have periodic group sessions to contemplate *worst case* scenarios to determine and make appropriate provisions.

4. *Quantifying operational and strategic risks.*

Compared to hazard and financial risks, this consideration remains a major challenge for many organizations because of the lack of models and/or historical data.

One solution is to attempt a qualitative analysis of operational and strategic risks and to determine the most significant ones. As well, causal models could be used to quantify the risks.

5. *Determining interrelationships between various risks.*

The process is difficult for hazard and financial risks for three primary reasons:

1. Past relationships between risks are not necessarily the same for the future.
2. Differences in time horizons.
3. As the number of risks to consider increases, the number of correlations to consider may become astronomical and too difficult.

Clearly, the process is even more difficult for operational and strategic risks, which are less quantifiable.

Therefore, one solution is to build models over time—adding components as new information becomes available.

6. *Incomplete risk transfer options.*

The options are limited for operational and strategic risks.

For hazard and financial risks, the risk transfer options currently include insurance and capital markets. However, organizations may require very specific ways to transfer risk not yet provided in those markets. Therefore, ongoing development of new products is required.

7. *Ongoing monitoring of ERM.*

ERM should be an on-going as opposed to one-time task. Regular reports and comparisons to prior risk assessments are necessary to ensure the necessary amendments to ERM are done. Changes to the risk environment are mainly beyond an organization's control, and so the organization needs to amend its approaches to handle risk.

8. *Gradual process.*

Basically, the idea is to start small, demonstrate initial success, and then move on to larger projects. That would help to secure interest and the *buy in* of ERM from existing and potential participants in the process.

KEY CONCEPTS

1. ERM involves a holistic and integrated approach to risk management as opposed to managing individual risks within an organization. Also, ERM considers the opportunities available in terms of organizations seeking to increase profitability.
2. Hazard risks (i.e., weather, injury, liability); financial risks (i.e., price, liquidity, credit, inflation, hedging); operational risks (i.e., general, management competence, information technology); and strategic risks (i.e., reputation, competition, change, innovation/obsolescence) are the four main types of risks addressed by ERM.
3. Solvency-related risk measures (i.e., shortfall risk, probability of ruin, VaR) deal with the negative side of the probability distribution with applications to economic capital requirements. Performance-related risk measures (i.e., variance, standard deviation, semi-variance) deal with the middle section of the probability distribution with applications to monitoring of volatility.
4. Two major stochastic models are used in practice. Statistical analytic models are relatively simple and use analytic methods. However, they do not take into account cause/effect relationships. In contrast, structural simulation models (Monte Carlo) are more complex and attempt to model real-life situations. They do take into account cause/effect relationships.
5. Generic applications of risk management concepts include optimization and candidate analysis. Specific applications include capital management, performance measurement, investment strategy/asset allocation, insurance/reinsurance/hedging strategy optimization, crisis management, contingency planning, business expansion/contraction strategy, distribution strategy, and strategic planning.
6. General points to consider for ERM implementation include: appointing an appropriate ERM leader; integrating ERM into an organization's culture; researching and gathering all potential risks in the organization; quantifying operational and strategic risks; determining interrelationships between various risks; incomplete risk transfer options; ongoing monitoring of ERM; and developing ERM gradually.

CONCEPT CHECKERS

1. The risk of a competitor developing a better product is most appropriately classified as which of the following types of risk?
 - A. Financial risk.
 - B. Hazard risk.
 - C. Operational risk.
 - D. Strategic risk.
2. Which of the following measures generally considers the fewest events but with the highest loss severity?
 - A. Economic cost of ruin.
 - B. Shortfall risk.
 - C. Tail value at risk.
 - D. Value at risk.
3. Which of the following attributes is not a characteristic of statistical analytical models?
 - A. Easy to parameterize.
 - B. Can mimic almost any situation contemplated.
 - C. Do not consider cause-and-effect relationships.
 - D. Calculation technique involves closed form solutions.
4. Which of the following decision-making applications of risk management concepts is best described as considering only a certain number of options, and the best of those are selected after a simple comparison to one another on an absolute basis in terms of risk/return?
 - A. Candidate analysis.
 - B. Capital management.
 - C. Investment strategy/asset allocation.
 - D. Optimization.
5. Which of the following points to consider regarding ERM implementation is the least difficult to implement?
 - A. Quantifying operational and strategic risks.
 - B. Integrating ERM into the organization's culture.
 - C. Determining interrelationships between various risks.
 - D. Researching and gathering all potential risks in the organization.

CONCEPT CHECKER ANSWERS

1. **D** Competition is an example of a strategic risk. In contrast, hazard risks involve issues such as fire, weather, injuries, and liability. Financial risks involve issues such as price, liquidity, credit, inflation, and hedging risks. Operational risks tend to involve issues relating to the firm internally (i.e., inefficient operations or incompetent management), while strategic risks often involve external parties.
2. **C** Tail value at risk is similar to ECOR because it considers the probability and severity of ruin. However, the former considers the severity of all events beyond the tail threshold amount—there are fewer of such events, but they have a much greater loss severity. Shortfall risk and value at risk explicitly do not consider events beyond the tail threshold amount.
3. **B** Statistical analytical models are simple and not designed to handle complex situations that can mimic almost any situation contemplated. Models with such abilities are called structural simulation models (Monte Carlo). All of the other attributes are characteristics of statistical analytical models.
4. **A** Candidate analysis is specific example of optimization, whereby only a certain number of options are considered and the best of those are selected after the analysis. Optimization, in general involves a mathematical process that selects the most appropriate option based on constraints, a probability distribution of uncertain conditions, and a particular amount to be maximized (i.e., return) or minimized (i.e., risk). A good example of optimization would be its use in investment strategy/asset allocation. Capital management generally involves determining optimal amounts of capital to be attributed and allocated to various business segments, so it is more likely to involve optimization, not candidate analysis.
5. **B** Although there may be some initial difficulties with integration of ERM into an organization's culture, the tangible benefits of ERM (i.e., cost savings, better information) could be presented to the various department heads with some reasonable probability of acceptance.

There are many risks to an organization that are difficult to identify or not even considered given their obscurity (i.e., environmental and terrorism threats).

Compared to hazard and financial risks, quantifying operational and strategic risks remains a major challenge for many organizations because of the lack of models and/or historical data.

Determining the interrelationships between various risks is difficult because past relationships between risks are not necessarily the same for the future because of differences in time horizons, and because as the number of risks increase, the number of correlations to consider may become too difficult to manage.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

CREATING VALUE WITH RISK MANAGEMENT

Topic 8

EXAM FOCUS

The goal of risk management is not to minimize or eliminate risk, but to increase the value of the firm. In previous topics, we examined risk reduction in the context of perfect/frictionless markets. In this topic, we look at the benefits of risk reduction when taxes and transactions costs exist and when economic agents do not always have identical information about the firm's prospects. For the exam, focus on the situations where risk reduction strategies can increase firm value.

REDUCING BANKRUPTCY COSTS

AIM 8.1: Explain how risk management can create value by handling bankruptcy costs.

When a firm has (risky) debt in its capital structure, there is some probability that the firm's operating income will be insufficient to pay the debtholders. In this case the firm may file for bankruptcy. A bankruptcy filing will further reduce the value of the firm as lawyers and advisers are hired and management time and energy must be devoted to the bankruptcy proceeding. Estimates by financial researchers of the average direct costs of bankruptcy are close to 3% of firm asset values (Weiss 1990).¹ The probability of filing bankruptcy times the costs incurred can be considered the expected **bankruptcy costs** of a firm that issues risky debt (debt for which there is some uncertainty about the firm's ability to satisfy the creditor claims).

More broadly, a firm with operating results that make bankruptcy a real possibility may experience costs of **financial distress**. Even if bankruptcy is avoided, a firm in financial trouble may experience added costs from debt renegotiation, forgone value-creating projects, stricter terms from suppliers, lost sales from the fear that the company will not be around to support their products, and the management time and energy devoted to these problems and related cash flow management issues.

The risk of bankruptcy and financial distress cannot be hedged by the shareholders as beta risk and output-price risk can; thus, it may be value increasing for the firm to undertake risk management to reduce or eliminate these costs. If risk management were costless, the firm would certainly eliminate the decrease in firm value due to potential bankruptcy and financial distress costs. Since hedging is a costly activity, the firm will employ risk management to reduce these costs as long as the increase in value from the reduction is

1. Weiss, Lawrence A. 1990. Bankruptcy resolution: Direct costs and violation of priority of claims. *Journal of Financial Economics*. 27(2): 285–314.

greater than the cost of hedging these risks. Through this use of risk management or hedging strategies, the value of the firm can be increased. In a *frictionless market*, risk management does not add to shareholder value; however, in a market with taxes, transaction costs, and agency costs, risk management can create value.

REDUCING THE VOLATILITY OF TAXABLE INCOME

AIM 8.2: Explain how risk management can create value by moving income across time and by reducing taxes.

When higher firm income is taxed at a higher *rate* than lower firm income, there is a possible reduction in total taxes from smoothing taxable income through risk management. Although income averaging is not allowed, losses in any period can be used to recover taxes paid in prior periods (**tax loss carrybacks**) or to reduce the tax liabilities in future periods (**tax loss carryforwards**). Tax loss carryforwards and carrybacks make no adjustment for the time value of money. Recovered taxes from past years have been out of the firm since being paid, and any reductions in future taxes due to current losses are not received until some future date.

There are many complicating factors in the corporate tax code, including the tax shield effects of interest paid and of depreciation taken. Even with all these complications, simulation analysis indicates that reducing the variability of taxable income through risk management does reduce the present value of taxes paid, therefore increasing the value of the firm. In general, risk management strategies that move a dollar of taxable income from a high tax year to a lower tax year can increase the value of the firm by reducing total taxes.

REDUCING THE WEIGHTED AVERAGE COST OF CAPITAL

In the tradeoff model of **optimal capital structure** (optimal proportion of debt financing), the tax benefits of additional debt increase firm value as more debt is used in financing the firm, up to the point where the increase in the potential costs of financial distress outweighs the additional tax savings from employing more debt. By decreasing the potential costs of financial distress, risk management can increase the optimal debt-equity ratio and increase the value of the firm by reducing the firm's overall funding costs (weighted average cost of capital).



Professor's Note: In the weighted average cost of capital (WACC) calculation, the contribution of debt is after-tax, hence the tax benefits of adding debt to a firm's capital structure.

REDUCING DIVERSIFIABLE RISK

AIM 8.3: Describe how risk reduction benefiting a large shareholder or stakeholder may increase or decrease firm value.

Previously, we argued that since investors can eliminate firm-specific (diversifiable) risk by holding a well-diversified portfolio of equities, there is no increase in firm value from risk management that hedges firm-specific risk. A large shareholder may benefit from a reduction in the firm's unsystematic risk, particularly if the shareholder's position in the firm represents a significant portion of his overall wealth. In other words, an investor may be more willing to hold a large position in a firm's stock when the diversifiable/firm-specific risk is lower.

The link to firm value arises because the firm may benefit from having a shareholder with a large position in the firm's equity. This benefit is derived from the fact that the large shareholder may have expertise in the firm's business and/or the firm's industry and can, therefore, provide advice that will help the firm's managers increase firm value. The existence of a large shareholder may also increase firm value due to the large shareholder's greater incentive to monitor management and influence management decisions, which prevents managers from taking actions that benefit management but do not increase firm value. That is, monitoring by a large shareholder can decrease **agency costs** that result from a divergence between management incentives and shareholder wealth maximization. Shareholders with relatively small positions in the firm have less incentive to devote the time and energy to monitoring, and they have less influence on the firm's management.

IMPROVING MANAGEMENT INCENTIVES

AIM 8.4: Explain the relationship between risk management, managerial incentives, and the structure of management compensation and its effect on firm value.

Another way to reduce the agency costs that can arise when manager self-interest and shareholder wealth maximization are not well aligned is to construct management compensation contracts that bring these two interests closer together. Commonly, bonuses based on some measure of firm or stock price performance are used to accomplish this. One of the problems with incentive compensation arrangements based on measures of firm performance is that some elements of firm performance are not under the control of management. An increase in the price of key inputs, a weakened market for the firm's products, or a general decrease in stock prices can all negatively affect incentive compensation, but they are not direct results of management actions and strategies.

Risk management that decreases nonmanagement-related risk factors in incentive compensation contracts can improve the incentives, thus increasing firm value. Further, incentive compensation that has less risk from factors outside management control can be written at a lower level of (expected) compensation, which also increases firm value.

Structuring management compensation contracts with incentive stock options can have a negative effect on firm value because of the decreased incentive for management to employ

risk management to increase value through tax saving or reduction of potential financial distress costs. Since the value of incentive stock options increases with increased volatility of outcomes, managers may choose less hedging through value-creating risk-management strategies than is optimal.

In addition to management incentives, others who have a stake in the firm's performance (i.e., stakeholders), such as employees, suppliers, and customers, can have better incentives to invest in firm-specific capital when the firm can hedge the risks inherent in such investment at a lower cost than they can. As an example, consider a supplier who invests in developing better parts or machinery for the firm. To the extent that the value of this investment is specific to the firm in question, the success of the firm can significantly affect the value of the investment. By hedging risk, the firm can offer better incentives for the supplier to make such an investment and, in the process, get better prices from the suppliers than they could offer if the supplier had to bear more risk related to the firm's performance over time. A similar argument can be made for employee investment in human capital specific to the firm and for commitments by customers that make their success more dependent on the firm's future performance.

REDUCING THE PROBABILITY OF DEBT OVERHANG

AIM 8.5: Describe debt overhang, and explain how risk management can increase firm value by reducing the probability of debt overhang.

Firms are said to have decreased value due to **debt overhang** when the amount of debt prevents equityholders from investing in positive net present value projects because the benefit to debtholders reduces the value created for equityholders. In this situation, managers fail to engage in projects that would increase the total value of the firm because financing these profitable projects by issuing equity would actually *decrease* the value of existing equity.

To better understand the implications of debt overhang, consider a firm that has experienced poor operating results to the point where there is significant probability that there will not be enough firm value to satisfy the debt obligations and to where the equity is worthless. One implication of this situation is that managers may accept high-risk projects that will decrease expected firm value but will also increase the probability of positive equity value at the end of the period. If the project outcome is poor, debtholders will bear the cost, and if project outcomes are good, equityholders will reap much of the benefit. Thus, debt overhang can cause managerial decisions that are detrimental to firm value.

If the leverage of the firm is so high that additional debt financing is not possible, management may also negatively impact firm value by not accepting positive net present value (value-increasing) projects. In this situation, financing a profitable project by issuing new equity will increase firm value, but most of the increase in value will accrue to the debtholders. The small increase in the value of the equity can be too little to offer the required return to the new equity. This shortfall will be made up by a dilution of value of the claims of existing equityholders. In this situation, debt overhang decreases firm value by causing management to forgo value-increasing opportunities.

Since debt overhang can decrease firm value for both of these reasons, risk management that reduces the probability of getting into a situation of debt overhang can increase firm value when the hedging costs are less than the increase in value from reducing the probability of debt overhang.

REDUCING INFORMATION ASYMMETRIES

AIM 8.6: Explain how risk management can reduce the problem of information asymmetry and increase firm value.

The idea that insiders/firm managers possess better information about a firm's performance and prospects than outside investors is referred to as **information asymmetry**. Information asymmetry may affect the cost of raising capital in two ways. First, if the firm seeks outside capital to fund a growth opportunity, investors must rely on management estimates of the quality/profitability of the opportunity. Second, the extent to which firm performance is the result of poor management decisions or of factors outside management control is to some extent unclear to outside investors.

Risk management can reduce both of these problems, leading to lower capital costs (i.e., lower WACC) and increased firm value. In the case of a growth opportunity, risk reduction may make financing based on current assets/projects possible, leading to lower funding costs. As discussed earlier, the problem of evaluating firm management may be reduced by hedging risks outside of management control. This can increase the confidence of outside investors that firm results reflect management quality, reducing funding costs and increasing firm value. The return investors require on funds invested in a firm depends on investors' perceptions of the quality of firm management and how well management incentives are aligned with investors' interests in maximizing firm value.

KEY CONCEPTS

1. When bankruptcy and financial distress are costly, reducing risk can increase the value of the firm by reducing the present value of expected future costs of financial distress in an amount greater than the cost of the hedging strategy employed.
2. Because of the nature of the corporate tax code, reducing the volatility of taxable income can reduce a firm's tax liability and increase firm value.
3. The optimal amount of debt in the firm's target capital structure can be increased by risk-reduction strategies, leading to lower funding costs and increased firm value.
4. A large shareholder may be valuable to the firm so that risk-reduction strategies, which reduce the risk and required return of a large shareholder, can increase firm value.
5. Risk management can clarify the relation between managerial decisions/actions and firm value, leading to more efficient management incentive compensation schemes.
6. By reducing the probability that a firm will become over-leveraged, risk management can increase firm value by reducing the potential for conflicts between the interests of debtholders and the interests of equityholders and managers.
7. Risk-management strategies can increase firm value by reducing the problem of asymmetric information, thereby reducing the firm's cost of capital.

CONCEPT CHECKERS

1. Risk management to reduce the probability of financial distress:
 - A. always increases firm value.
 - B. can increase firm value because financial distress has measurable costs.
 - C. is easily replicated by individual shareholders.
 - D. cannot reduce the weighted average cost of capital.
2. Tax loss carrybacks and carryforwards do not negate the benefits of managing the volatility of taxable income because:
 - A. carryforwards are limited in amount.
 - B. income averaging is not allowed.
 - C. the time value of money is not accounted for.
 - D. corporations pay a flat tax rate above a certain amount.
3. Reducing the firm's diversifiable risk can increase firm value by:
 - I. decreasing the required rate of return on equity.
 - II. attracting large shareholders with related business experience.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.
4. A situation in which a profitable project is forgone because a significant portion of the net present value of the project will accrue to debtholders is called:
 - A. financial distress.
 - B. technical default.
 - C. agency costs.
 - D. debt overhang.
5. Risk management that reduces the problem of information asymmetry:
 - I. may reveal poor management decisions more clearly.
 - II. can reduce the weighted average cost of capital.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.

CONCEPT CHECKER ANSWERS

1. B Financial distress will take up management time and energy and possibly lead to stricter terms from suppliers and loss of customers. Therefore, reducing the probability of financial distress can increase firm value.
2. C A dollar of taxes avoided in the future because of current-period losses is not worth as much as a dollar today, and the return of a dollar of taxes paid in a prior period does not include interest for the time it has been out of the firm.
3. B Large shareholders can provide valuable monitoring of management and firm-specific expertise. A reduction in diversifiable risk will lower the risk of a portfolio with a large position in the firm's equity, making holding a large position more attractive.
4. D With debt overhang, managers fail to engage in projects that would increase the total value of the firm because financing these profitable projects by issuing equity would actually decrease the value of existing equity.
5. C Reduction in information asymmetry can lower the cost of capital by increasing management credibility about new growth opportunities. It may also reveal poor management decisions more clearly.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

FINANCIAL DISASTERS

Topic 9

EXAM FOCUS

These cases illustrate a number of financial and operational risk management failures. Specifically, we will examine cases involving misleading reporting, large unexpected market movements, and inappropriate customer conduct. Pay close attention to the causes of these financial disasters and how they could have been prevented. You should be prepared to handle questions on these recurring themes.

AIM 9.1: Describe the key factors that led to and the lessons learned from the following risk management case studies:

- Chase Manhattan and their involvement with Drysdale Securities
- Kidder Peabody
- Barings
- Allied Irish Bank
- Long Term Capital Management (LTCM)
- Metallgesellschaft
- Bankers Trust

MISLEADING REPORTING CASES

The following cases demonstrate situations where investors, firms, and lenders were misled about the nature and size of investment positions. In all cases, the motivation to mislead was driven by the desire to make a large profit. The large potential gain was sought by taking large risky positions, thereby creating a severe moral hazard issue. The importance of independent risk management oversight is apparent after reading these cases.

Drysdale Securities and Chase Manhattan Bank

In 1976, Drysdale Government Securities was able to borrow \$300 million in unsecured funds from Chase Manhattan. The borrowed funds far exceeded the capital of Drysdale of \$20 million and consequently any amount it would have normally been approved for. The company used the borrowed funds to take bond positions, which eventually declined in value. Given the loss in market value, Drysdale was unable to repay the borrowed funds and was forced into bankruptcy. Almost all of the losses had to be absorbed by Chase Manhattan since it brokered most of Drysdale's borrowings.

Drysdale obtained these funds by exploiting a flaw in the market's system for computing the value of collateral of United States government bonds. In an effort to save time, this collateral was often valued without the consideration of accrued interest. This imprecise calculation method allowed Drysdale to take advantage of the difference between the cash

value borrowed securities could be sold for, which accounted for accrued interest, and the amount of cash collateral required to be posted against the borrowed securities, which did not account for accrued interest.

Clearly, misleading reporting was used by Drysdale in order to obtain the borrowed funds. However, Chase Manhattan was partially to blame for assuming that it was simply the middleman in the transactions and the positions taken had a low level of risk. The inexperienced managers at Chase failed to realize that the contract wording with Drysdale indicated that Chase would be held responsible for any payments due. This financial disaster convinced the securities industry to develop more accurate methods for computing collateral when borrowing bonds. The situation also led Chase and other firms to seek the approval of a risk control function when issuing new funds.

Kidder Peabody

The head of the government bond trading desk at Kidder Peabody, Joseph Jett, misreported a series of trades between 1992 and 1994, which allowed him to report substantial artificial profits. After these errors were detected, \$350 million in falsely reported gains had to be reversed. The series of events did not result in actual losses for the firm since the profits were fake; however, the trades triggered a loss of confidence in the management of Kidder Peabody, which was owned by General Electric (GE). GE ended up selling Kidder to Paine Webber, which ultimately dismantled the troubled company.

Jett was able to report false profits since the computer system used to report government bond trading activity did not account for a forward contract's present value. This enabled Jett to earn an instant profit when purchasing a bond for cash and delivering the forward contract. The system failed to realize that this profit would disappear once financing costs for the cash bond were taken into account.

Increasing the size of the forward contracts, as well as the length of the delivery period, enabled Jett to further exploit the computer's accounting error. Eventually, Jett profits came under fire after Kidder realized that no individual trading strategy could produce the substantial profits that were being reported. This misleading reporting case demonstrates the importance of investigating large profits from unknown trading strategies.

Barings

Nick Leeson, a British Barings junior trader in Singapore, took speculative derivative positions in an effort to recoup prior trading losses that he was able to hide fraudulently. The losses went undetected due to inadequate control systems.

In 1994, Leeson lost \$296 million through his trading activities, but reported a profit of \$46 million to management. His trading supposedly involved two main strategies—selling straddles on the Nikkei 225 and arbitraging price differences on Nikkei 225 futures contracts that were trading on different exchanges. A short straddle strategy involves selling calls and puts. It is profitable when the underlying index remains relatively unchanged over the life of the straddle, in which case the calls and puts expire worthless, leaving the option writer with the option premiums. The Nikkei 225 futures arbitrage involves taking a long

futures position on one exchange where the price is relatively low and hedging with an offsetting short position on another exchange where the price is relatively higher.

Leeson had previously incurred huge trading losses that would have cost him his job if they were revealed. In an effort to recover those losses, he abandoned the hedged posture in the long-short futures arbitrage strategy and initiated a speculative long-long futures position on both exchanges in hope of profiting from an increase in the Nikkei 225. This move exposed the firm to enormous market risk and event risk, which stems from unexpected major events that, while not directly related to markets, can affect markets.

On January 17, 1995, an earthquake hit Japan. The Nikkei plunged, creating huge losses on both the short straddle and the double-long futures position. The resulting margin calls were satisfied for a time because in 1994, Leeson had requested and received without question \$354 million from the London office because they believed his strategy was riskless. This lack of oversight contributed to Barings' failure as the Nikkei continued to drop. Between 1993 and 1995, Leeson's actions resulted in losses of approximately \$1.25 billion and forced Barings into bankruptcy.

In addition to being Barings' floor manager on the Singapore International Monetary Exchange (SIMEX) trading floor, Leeson was in charge of settlement operations. This position allowed him to influence back-office employees to hide his trading losses from the London office. He was able to hide speculative positions by reporting these positions for fictitious customers. He used an old error account to book losing trades for these fictitious customers and used his back-office influence to prevent that trading activity from being reported to the main office in London.

To book profits that would be reported to London, Leeson initiated cross trades on the SIMEX in which the same firm buys and sells a security at the current market price. Again using his back-office influence, he directed settlement employees to modify the execution price, making one side of the trade profitable and the other unprofitable. The profitable trade was booked to the standard trading accounts, which were reported to management, while the unprofitable trade was booked to the old error account that escaped reporting to senior management. By incorrectly booking these losses, Leeson was able to report substantial profits in 1994, which allowed him to earn a \$720,000 bonus.

Leeson was able to illegally book fraudulent trades because there was *little management oversight of the settlement process*. Leeson was responsible for reporting to multiple managers in a convoluted organizational structure. This situation created ambiguity concerning who was responsible for performing specific oversight functions. In addition, political power struggles and senior management's lack of understanding about Leeson's role eroded oversight and allowed trading losses to be hidden.

Officially, Leeson was subject to risk controls that limited the amount of speculative trades he was allowed to make. In practice, however, he ignored and vastly exceeded those limits. These violations went undetected because Barings lacked risk management oversight that would have monitored positions, strategies, and risk. This oversight was so poor that the London office transferred \$354 million to meet margin calls without questioning Leeson. If management had a better understanding of Leeson's trading strategies, they would have recognized that his reported profits were disproportionate to the purported riskless trading. In sum, weak management at Barings failed to establish information, reporting, and control

systems. If trading and settlement responsibilities rested with different people, coordinating the trading and settlement schemes required to hide trading losses would be much more difficult. It would have created a system of checks and balances that might have prevented Leeson from taking wildly speculative positions.

Allied Irish Bank

Between 1997 and 2002, a currency trader for Allied Irish Bank (AIB), John Rusnak, hid \$691 million in losses from management. Rusnak used a number of deceptive means to hide these losses including bullying back-office workers into not following-up on trade confirmations for imaginary trades. However, in 2001, the back-office supervisor realized that something was amiss when he saw that confirmations were missing for a number of trades. After this problem was corrected, the fraudulent actions were eventually identified.

Management believed that Rusnak was running a small currency arbitrage trading strategy. However, the strategy actually being implemented involved very large currency positions. Rusnak was able to hide these trading activities from management by creating imaginary trades to offset his real trades. The result was the appearance of a trading strategy that involved small positions. Rusnak made a point of only reporting modest gains as to not raise any red flags. For a time, he was able to cover losses by selling deep in the money options, which provided him with large option premiums. He further disguised his actions from management by entering false positions in the firm's system for calculating risk measures such as value at risk (VaR).

AIB's management was partially to blame for its inexperience in being unable to figure out Rusnak's trading activities. Suspicious trades and trading profits were ignored by management as Rusnak continually manipulated the firm's operations staff. For a time, Rusnak even forged trade confirmations after the back-office supervisor realized the incorrect actions. This case is similar to the actions that led to the bankruptcy of Barings. However, Rusnak did not have the advantage of Leeson of also running the back-office operations. Instead, Rusnak used his strong personality to bully back-office employees and took advantage of the fact that trades were being transacted in the over-the-counter market, which did not require immediate cash settlement.

Union Bank of Switzerland

During 1997, Union Bank of Switzerland's (UBS) equity derivatives business lost between \$400 and \$700 million. An additional loss of \$700 million followed the next year, which was mostly due to its large stake in Long-Term Capital Management (LTCM). Losses at UBS forced the firm to merge with Swiss Bank Corporation (SBC).

It is unclear which trades directly influenced the losses at UBS, but it is accepted that the losses resulted due to inadequate actions on the part of the firm's risk controllers. The equity derivatives business at UBS was being run with an unusual amount of independence. The department's senior risk manager was also the head of quantitative analytics, which enabled him to not only provide input for business decisions, but also have his compensation tied to trading results.

It is suspected that the losses in 1997 were due, in some part, to the following four factors: (1) British law tax changes; (2) large Japanese bank warrants, which were inappropriately hedged against a drop in the underlying stocks; (3) incorrect valuation of long-dated options on equity baskets; and (4) inappropriate modeling of other long-dated options. The suspected losses in 1998 were largely tied to the failure of LTCM. UBS's exposure to LTCM involved a 40% direct investment in the hedge fund and a 60% exposure to written options on the fund. By taking these two positions, UBS was hoping to delta-hedge their exposure to LTCM; however, LTCM's lack of transparency made it difficult for UBS to fully understand the nature of its positions. It was believed that UBS failed to properly analyze and stress test its positions.

Daiwa

A Treasury bond trader, Toshihide Iguchi, covered up \$1.1 billion in losses over an 11-year time span from 1984 to 1995. Iguchi was able to not only hide these losses, but also forge customer trading slips, which actually made his actions appear profitable to Daiwa Bank's management. This misleading reporting went undetected due to Iguchi's dual role as the head of both trading and the back-office support function. When senior executives finally learned of the fraud, they failed to promptly report it to the authorities. As a result, Daiwa lost its trading license in the United States.

Sumitomo

Yasuo Hamanaka, the lead copper trader for Sumitomo, attempted to corner the copper market in a classic market manipulation strategy. Because the copper market was relatively small, Hamanaka had the potential to control and corner it.

He essentially established a dominant long position in futures contracts and simultaneously purchased large quantities of physical copper. As the futures contracts approached delivery, the party with the short position would find little physical copper available for delivery and would be forced to either pay a large premium for physical copper or unwind its short position by taking an offsetting long futures position. Either way, the price of copper and/or copper futures prices would rise and create handsome profits for Hamanaka and Sumitomo.

The risk, of course, was that severe losses would be unavoidable if copper prices fell. Subjecting the firm to enormous market risk to help finance his long copper positions, Hamanaka sold put options, which exposed the trading strategy to the risk of falling copper prices even more.

Hamanaka's unusually low degree of supervision and broad powers allowed him to implement this fraudulent trading strategy without detection, until the Commodity Futures Trading Commission (CFTC) began an investigation of market manipulation in December of 1995. The CFTC's interest was piqued by the possibility that Sumitomo had purposely influenced the price of copper with positions that were unrelated to legitimate commercial needs, a critical element in the determination of market manipulation.

In May of 1996, Hamanaka was reassigned to another position, sparking suspicion among other copper traders who began to sell their copper holdings in anticipation of Sumitomo doing the same. A continuation of plummeting copper prices resulted in a \$2.6 billion trading loss and a \$150 million fine from the CFTC. Hamanaka was fired, prosecuted,

and jailed. The size of Sumitomo's copper positions in relation to the size of the market exacerbated the drop in copper prices.

Sumitomo's lack of supervision on Hamanaka created a high degree of operational risk, which could have been reduced with proper internal controls. For example, because Hamanaka had almost total autonomy, he was able to give power of attorney to brokerage firms to execute highly leveraged transactions in a scheme to help finance his accumulation of copper. In addition, the lack of supervision allowed him to keep two sets of trading books, one of which reported large profits. The other set recorded huge losses and was secret, which allowed the illegal activities to go undetected.

Large transactions should have required multiple approvals by senior management, who would have an understanding of the trading strategy. In Sumitomo's case, however, no approvals were necessary, and senior management was unequipped to understand the complex transactions.

Askin Capital Management and Granite Capital

David Askin managed both the Askin Capital Management and Granite Capital hedge funds, which invested in mortgage securities. He misled investors by valuing positions with incorrect values instead of dealer quotes. Askin reported these incorrect values to potential clients in order to generate interest in his funds. Both funds went bankrupt in 1994, suffering losses of \$600 million.

Merrill Lynch

In 1987, Merrill Lynch reported losses of \$350 million from its trading in mortgage securities. The losses resulted from a mistake in the firm's calculation of duration. The firm was using a 13-year duration calculation for 30-year mortgages, which is generally correct when considering all interest and principal payments. However, since Merrill Lynch was selling the interest-only portion of the mortgage securities, the correct duration was actually more in-line with the duration of the principal-only portion, which was 30 years.

National Westminster Bank

National Westminster Bank's (NatWest) traders used incorrect volatility inputs for interest rate caps and swaptions between 1994 and 1997. It was reported that traders were only using a sample of market volatility estimates due to the illiquid nature of these investments. The loss from this incorrect reporting was close to \$140 million. NatWest was forced to sell the Royal Bank of Scotland due to investor's loss of confidence in management's oversight.

LARGE MARKET MOVEMENT CASES

The following two cases on Metallgesellschaft and Long-Term Capital Management illustrate financial disasters related to large unexpected market movements. Unlike the previously discussed cases, misleading positions were not the cause of the substantial losses. These two cases share many common financial themes, including an extreme lack of liquidity.

Metallgesellschaft

In 1991, Metallgesellschaft Refining and Marketing (MGRM), an American subsidiary of Metallgesellschaft (MG), an international trading, engineering, and chemicals conglomerate, implemented a marketing strategy designed to insulate customers from price volatility in the petroleum markets for a fee.

MGRM offered customers contracts to buy fixed amounts of heating oil and gasoline at a fixed price over a 5- or 10-year period. The fixed price was set at a \$3 to \$5 per barrel premium over the average futures price of contracts expiring over the next 12 months. Customers were given the option to exit the contract if the spot price rose above the fixed price in the contract, in which case MGRM would pay the customer half of the difference between the futures price and contract price. A customer might exercise this option if she did not need the product or if she were experiencing financial difficulties. In later contracts, the customer could receive the entire difference in exchange for a higher fixed contract price.

The customer contracts effectively gave MGRM a short position in *long-term forward contracts*. MGRM hedged this exposure with long positions in *near-term futures* using a *stack-and-roll hedging strategy*. In this strategy, the firm buys a bundle of futures contracts with the same expiry date, known as a *stack*. Just prior to delivery, the firm liquidates the stack and buys another stack of contracts with longer expirations, known as a *roll*. The level of uncertainty in the cost of this strategy should have prompted MGRM to use a valuation reserve since they were currently basing roll costs on historical data rather than potential future costs.

MGRM used short-term futures to hedge because alternatives in the forward market were unavailable and long-term futures contracts were highly illiquid. As it was, MGRM's open interest in unleaded gasoline contracts was 55 million barrels in the fall of 1993, compared to average trading volume of 15 to 30 million barrels per day. In December of 1993, MGRM cashed out its positions and reported losses of approximately \$1.5 billion.

Although some market observers cite the maturity mismatch between MGRM's short position in long-term fixed-rate contracts with customers and its long position in near-term futures contracts, many economists believe this hedging strategy is fundamentally sound. Over the life of a properly constructed hedge, the cash flows from the forward and futures contracts would balance out, provided the hedging firm could withstand interim cash flow requirements from marked to market losses, margin calls, credit risks, and liquidity risks associated with adverse market movements. The fundamental issue for MGRM was a cash flow problem that constrained the company's ability to ride out the hedge. This cash flow problem had several causes and severe consequences, which are discussed next.

Gains and losses on forward contracts are realized at the agreement's expiration, whereas futures contracts are marked to market such that the gains and losses are realized on a daily basis. In MGRM's case, gains and losses on its customer contracts were realized if and when the customers took delivery, which would occur over a 5- to 10-year period.

During 1993, oil prices dropped from a high of about \$21 per barrel to about \$14 per barrel, resulting in losses of \$900 million on MGRM's long positions, which were realized immediately as the futures contracts were marked to market. The offsetting gains on their customer contracts, however, would not be realized for years to come, which created

potential short-term cash outflows, and resulted in **funding liquidity risk**. Declining oil prices also created margin calls that exacerbated the cash flow problem. Due to these losses, MG ordered MGRM to close out of its customer contracts. This forced the firm to unwind its positions at very unfavorable terms.

According to German accounting rules, MGRM was required to report losses associated with its futures hedges, but was not permitted to show associated gains from its customer contracts, which the futures were meant to hedge. The magnitude of the losses caused its credit rating to drop, increasing its perceived credit risk and restricting the company's access to credit. The losses also created a crisis of confidence with its counterparties, which began to suspect the firm was speculating rather than hedging and, therefore, demanded collateral to secure contract performance. These same concerns induced the New York Mercantile Exchange to increase the firm's margin requirements. It is interesting to note that these consequences, which aggravated an already mounting cash flow problem, did not stem from a fundamental flaw in MGRM's hedging program. They occurred due to overly conservative financial reporting requirements that failed to recognize the relationship between hedging losses and offsetting gains on the underlying positions that motivated the hedge in the first place.

The cash outflows might have been tolerable and possibly balanced out by cash inflows over the life of the hedge were it not for the sheer size of MGRM's position, which would have taken ten days to liquidate. To liquidate without affecting market prices would have taken from 20 to 55 days. As a result, the company lacked liquidity to unwind its positions, if necessary, without significant market impact, and was therefore subject to **trading liquidity risk**. To make matters worse, MGRM was carrying a heavy debt load and had little equity to withstand losses and cash flow problems on positions of this size.

Long-Term Capital Management

Long-Term Capital Management (LTCM), a hedge fund founded in early 1994, generated stellar returns in its first few years of operation: 43% in 1995 and 41% in 1996. The partners worked together at Salomon Brothers (now Citigroup) and, given their success, decided to start their own fund and proceeded to seek capital from investors. Funding was provided to LTCM despite the secretive nature of its positions. In addition, investors were locked into investments for long periods of time in order to prevent liquidation issues since the fund was focused on long-term investment strategies. In the later years of operations, the partners at LTCM invested a large portion of their net worth in the fund since they believed so strongly in the success of their trading strategies.

With positions in equity, fixed income, and derivatives markets all around the globe, LTCM grew enormously. At the beginning of 1998, it had \$125 billion of assets on \$4.7 billion of equity capital, yielding leverage of 28 to 1. Although this **balance sheet leverage** was in line with other large investment banks, it underestimated the true leverage by overlooking the **economic leverage** in LTCM's positions. For example, LTCM's positions represented notional principal in excess of \$1 trillion. The astronomical use of leverage was possible because financial institutions often waived initial margin requirements based on the reputation of the principals, freeing up capital to take on more leverage.

Most of LTCM's investment strategies could be classified as relative value, credit spreads, and equity volatility. Their relative value strategies involved arbitraging price differences among similar securities and profiting when the prices converged. One benefit of this convergence strategy is that being long and short similar securities hedges risk exposure and reduces volatility.

LTCM believed that, although yield differences between risky and riskless fixed-income instruments varied over time, the risk premium (or credit spread) tended to revert to average historical levels. Noticing that credit spreads were historically high, they entered into mortgage spreads and international high-yield bond spreads intending to profit when the spreads shrank to more typical historical levels. Similarly, their equity volatility strategy assumed that volatility on equity options tended to revert to long-term average levels. When volatility implied by equity options was abnormally high, LTCM "sold volatility" until it regressed to normal levels.

In August of 1998, Russia unexpectedly defaulted on its debt, sending Russian interest rates soaring to 200% and crushing the value of the ruble. This economic shock triggered investor concern about already faltering economies in the Pacific rim, causing the yields on developing nations' debt to increase and a flight to the quality of government bonds in industrialized countries. Yields on corporate debt—both high and low quality—also increased sharply. In other words, the flight to quality increased, rather than decreased, credit spreads, causing huge losses for LTCM. Shortly thereafter, Brazil also devalued its currency, thereby further increasing interest rates and risk premiums. The general increase in volatility also generated losses in LTCM's equity volatility strategies.

Although prices in relative value arbitrage strategies sometimes diverge and create temporary losses before they ultimately converge, the large increase in yield spread caused huge losses and severe cash flow problems caused by realizing marked to market losses and meeting margin calls. The effect of the losses and the cash flow crisis were compounded by the firm's hyper leverage. LTCM lost 44% of its capital in just one month. The firm's lack of equity capital created a cash flow crisis and made it necessary to liquidate positions to meet margin calls.

If LTCM had sufficient equity to withstand the cash flow crisis created by the sharp divergence of asset prices, it might have ultimately been able to realize the benefits of convergence. Instead, LTCM risked the possibility of insolvency before convergence could occur. Notice the similarity to the funding liquidity risk in the Metallgesellschaft case.

One of the fundamental risks faced by LTCM was model risk, the risk that valuation or trading models are flawed. Their models assumed that historical relationships were useful predictors of future relationships, which is often true in the absence of economic shocks. However, external shocks often cause correlations that are historically low to increase sharply. When Russia defaulted on its debt, credit spreads, risk premiums, liquidity premiums, and volatility around the world increased. LTCM partly adjusted for this possibility by using correlations that were greater than historical correlations in their stress tests. However, these adjustments inadequately captured the spike in correlations caused by the cascading effect of economic shocks.

The models also assumed that low-frequency/high-severity events were uncorrelated over time. Rather than occurring highly infrequently and independently over time, one economic shock triggered another so that extremely low probability events were occurring several times per week. As a result, traditional VaR models underestimated risk in the tails of the distribution.

LTCM was diversified across the globe, across different asset classes, and across different trading strategies. Fundamentally, however, all of its trading strategies were based on the notion that risk premiums and market volatility would ultimately decline. Since the success of all its trading strategies hinged on a single economic prediction, LTCM was far less diversified than a cursory exam would suggest and was, therefore, subject to market risk.

LTCM's extreme leverage enabled it to assume extremely large, high-profile positions that attracted the attention of imitators who initiated similar or identical trades, thereby adding to the size of LTCM's positions in some sense. When it became necessary to liquidate positions, the firm found itself in the position of being a market maker, rather than a price taker as traditional valuation models assume. In addition to suffering the price impact of liquidating its own enormous positions, LTCM found itself competing with imitators who were also liquidating their positions. Market prices largely depended on expectations about LTCM's actions.

Falling prices resulting from LTCM's forced liquidation created more marked to market losses and margin calls, which forced more liquidations that resulted in a self-reinforcing cycle. LTCM considered the possibility of market impact to some extent in its short risk measures, but underestimated the magnitude of its influence on market prices, particularly in the event of forced liquidation. Trading liquidity risk was also present in the Metallgesellschaft case.

As a hedge fund, LTCM's reporting obligation to regulators was limited. Although the size of its positions required financial statement reporting and daily position reporting, these reports were incomplete and lacked disclosure of derivative positions and trading strategies. Ultimately, the Federal Reserve Bank of New York orchestrated a bailout in which 14 leading banks and investment houses invested \$3.65 billion for a 90% stake in LTCM.

The LTCM case demonstrated the need for several suggested improvements when implementing risky investment strategies and seeking investor funds. One suggestion is to ensure that an initial margin is provided. LTCM had to mark their positions to market, but in many cases, the initial margin was waived. Another suggestion is to incorporate potential liquidation costs into prices in the event of adverse market conditions. A third suggestion is the need for greater position disclosure. A final suggestion is better utilization of stress testing when evaluating financial risk; namely credit risk. LTCM planned for the possibility of increasing disruptions in short-term market movements. However, it failed to supplement VaR measures with stress scenarios that incorporated the possibility that competitors were holding similar positions that might be liquidated at the same time in the event of extreme market movements.

CUSTOMER CONDUCT CASES

These cases describe actions that led to significant decreases in firm reputation among its customers (i.e., reputational risk). The actions relate to misleading investors on the risk of certain investments. Failure to perform the necessary due diligence subjected customers to huge losses which were, in some cases, followed by fines and settlements for the firms involved. The actions themselves, however, did not create direct losses for the firms.

Banker's Trust

Procter & Gamble (P&G) and Gibson Greetings sought the assistance of Banker's Trust (BT) to help them reduce funding costs. BT used derivative trades, which promised the two companies a high-probability, small reduction in funding costs in exchange for a low-probability, large loss. Unfortunately, the derivative trades only resulted in significant losses for both P&G and Gibson.

The derivative structures developed by BT were intentionally complex and prevented P&G and Gibson from fully understanding the trade values and risks that were involved. In addition, the structures were not comparable to other company derivative trades making it impossible to get a competitive quote. P&G and Gibson were further misled into thinking that the structures were tailored to meet their individual needs. In 1994, P&G and Gibson finally realized that they had been misled after discovering that they had suffered huge losses. As a result, the two companies sued BT.

It was common practice for BT to tape phone conversations of its traders and marketers in an effort to resolve possible verbal contract disputes. Unfortunately for BT, these tapes were used as evidence during the lawsuit since they picked up internal conversations regarding the derivative structures in question. In some of these conversations, BT's staff bragged about how badly they fooled clients with complex structures and showed how price quotes given to P&G and Gibson were manipulated.

The Banker's Trust scandal severely damaged its reputation and forced its CEO to resign. BT was eventually acquired by Deutsche Bank and ultimately dismantled. The actions at BT led to tighter controls for dealing with clients at other firms. This case demonstrated the importance of matching trades with a client's needs and providing price quotes that are independent from the front office. It also demonstrated the importance of exercising caution with any form of communication that could eventually be made public, as it could damage a firm's reputation if unethical practices are present.

Prudential-Bache Securities

Prudential-Bache Securities misled investors regarding the risk of investments in limited partnerships. The incorrect identification of risk impacted thousands of investors and was so severe that it resulted in over \$1 billion in fines and settlements.

Morgan Grenfell Asset Management

A fund manager at Morgan Grenfell Asset Management incorrectly directed investors into highly speculative equity investments. In addition, this manager found a way to bypass legal restrictions regarding the percentage of a stock that a mutual fund could hold at one time. In 1995, Morgan Grenfell was ordered to pay approximately \$600 million to investors to make up for losses incurred by incorrectly investing in speculative securities.

JPMorgan

JPMorgan misled Korean customers on the risk of certain derivative transactions. When the Thai baht exchange rate suffered a significant drop against the U.S. dollar in 1997, Korean customers lost hundreds of millions of dollars. As a result, the firm's reputation was damaged.

KEY CONCEPTS

1. Drysdale Securities was able to borrow \$300 million in unsecured funds from Chase Manhattan by exploiting a flaw in the system for computing the value of collateral.
2. The head of the government bond trading desk at Kidder Peabody, Joseph Jett, reported substantial artificial profits. After the false profits were detected, \$350 million in previously reported gains had to be reversed.
3. Hidden trading losses at Barings induced Nick Leeson to abandon hedging strategies in favor of speculative strategies. A lack of operational oversight and his dual roles as trader and settlement officer allowed him to conceal his activities and losses.
4. A currency trader for Allied Irish Bank, John Rusnak, hid \$691 million in losses. Rusnak bullied back-office workers into not following-up on trade confirmations for imaginary trades.
5. UBS's equity derivatives business lost millions in 1997 and 1998. The losses were mostly due to incorrect modeling of long-dated options and the firm's stake in Long-Term Capital Management.
6. Yasuo Hamanaka, a trader for Sumitomo, attempted to corner the copper market by buying large quantities of physical copper and taking a long futures position as well. A lack of operational and risk controls permitted the scheme to continue undetected until copper prices plunged, creating huge losses.
7. The financial crisis at Metallgesellschaft resulted fundamentally from cash flow timing differences associated with the positions making up its hedge. Cash flows on short forward contracts occurred over the distant future. Cash flows on long futures contracts occurred daily. In addition, the sizes of the positions were so large that it prevented the company from liquidating its positions without incurring large losses.
8. Extreme leverage, a lack of diversification, and inadequate risk models put Long-Term Capital Management in a cash flow crisis when an economic shock created intolerable marked to market losses and margin calls. A forced liquidation of its huge positions drove prices down, further compounding their losses.
9. Banker's Trust developed derivative structures that were intentionally complex and prevented Procter & Gamble and Gibson Greetings from fully understanding the trade values and risks that were involved. In taped phone conversations, BT's staff bragged about how badly they fooled clients.

CONCEPT CHECKERS

1. Which of the following was least influential in the Metallgesellschaft debacle?
 - A. Fraud.
 - B. Timing differences in the cash flows of its long and short positions.
 - C. The size of its positions influenced market prices.
 - D. Financial reporting requirements.
2. Which of the following financial disasters created a situation that resembled a classic Ponzi scheme where artificial profits are shown, but never materialize into actual profits?
 - A. Drysdale Securities.
 - B. Banker's Trust.
 - C. Kidder Peabody.
 - D. Merrill Lynch.
3. In 1997, equity derivative losses at the Union Bank of Switzerland (UBS) appeared to be related to four different factors. Of the factors shown below, which ones are most unique to UBS (i.e., did not impact competitors)?
 - I. British law tax changes and large Japanese bank warrants.
 - II. Incorrect valuation of long-dated options on equity baskets and inappropriate modeling of other long-dated options.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.
4. Hedging models at Long-Term Capital Management accounted for the:
 - I. spike in correlations among asset class prices during times of economic crisis.
 - II. dependence of catastrophic events through time during global economic shocks.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.
5. Nick Leeson's now infamous trading strategies in 1994 and 1995 at Barings Bank focused on calculated bets on the Nikkei 225. Which of the following trading strategies did not contribute to the staggering losses that ultimately forced Barings into bankruptcy?
 - I. Long-long futures arbitrage.
 - II. Long straddle.
 - A. I only.
 - B. II only.
 - C. Both I and II.
 - D. Neither I nor II.

CONCEPT CHECKER ANSWERS

1. A The fundamental problem at Metallgesellschaft was that the timing of the marked to market losses and margin calls on its futures contracts were mismatched with the cash flows on the forward contracts it was trying to hedge. The problem was compounded by the enormous size of the positions, which made liquidation costly, and by conservative financial reporting requirements that did not recognize the gains on the forward contracts. Neither fraud nor deception is central to the Metallgesellschaft case.
2. C The head of the government bond trading desk at Kidder Peabody, Joseph Jett, misreported a series of trades, which allowed him to report substantial artificial profits. After these errors were detected, \$350 million in falsely reported gains had to be reversed. This situation of hypothetical profits in place of promised profits resembles a classic Ponzi scheme.
3. B Statement I resembles factors that affected UBS as well as its competitors. The bank warrant positions may have been larger than its competitors, but they were not unique to UBS. Statement II resembles factors that were unique to UBS.
4. D The models used by LTCM primarily relied on historical correlations to measure risk. In doing so, the firm failed to account for the spike in correlations caused by economic shocks, such as Russia defaulting on its debt. The models also did not consider that infrequent shocks might be clustered in time, one causing another. As it happened, risk premiums rose across the globe, forcing LTCM to liquidate positions because its relatively miniscule equity basis was insufficient to withstand the losses. The size of its positions aggravated negative price trends that were already set in motion.
5. B After incurring huge trading losses, Leeson made an effort to recover those losses by abandoning his original hedged position in a long-short futures arbitrage strategy and initiated a long-long futures position on two trading exchanges. As well, one of his other trading strategies was selling straddles on the Nikkei 225 (which would have been profitable had the underlying index remained relatively unchanged).

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

RISK MANAGEMENT FAILURES: WHAT ARE THEY AND WHEN DO THEY HAPPEN?

Topic 10

EXAM FOCUS

Risk management failures result from not correctly recognizing, measuring, and/or monitoring risks as well as not appropriately communicating these risks to top management. Mismeasurement of risk can result from not recognizing how return distributions change, using subjective inputs concerning rare events, and failing to take all risks into account. For the exam, understand the use of value at risk (VaR) as a risk metric. VaR is a very useful tool for measuring and monitoring market, credit, and operational risk.

THE ROLE OF RISK MANAGEMENT

AIM 10.1: Define the role of risk management and explain why a large financial loss is not necessarily a failure of risk management.

The role of risk management involves performing the following tasks.

- Assess all risks faced by the firm.
- Communicate these risks to risk-taking decision makers.
- Monitor and manage these risks (make sure that the firm only takes the necessary amount of risk).

The risk management process focuses on the output of a particular risk metric [e.g., the value at risk (VaR) for the firm] and attempts to keep the measure at a specified target amount. When a given risk measure is above (below) the chosen target amount, the firm should decrease (increase) risk. The risk management process usually evaluates several risk metrics (e.g., duration, beta).

A large loss is not necessarily an indication of a risk management failure. As long as risk managers understood and prepared for the possibility of loss, then the implemented risk management was successful. With that said, the main objective of risk management should not be to prevent losses. However, risk management should recognize that large losses are possible and develop contingency plans that deal with such losses if they should occur.

INCORRECTLY MEASURING AND MANAGING RISK

AIM 10.2: Describe how risk management can fail.

AIM 10.3: Describe how risk can be mismeasured.

The process of risk management can fail if one or more of the following events occur.

- Not measuring known risks correctly.
- Not recognizing some risks.
- Not communicating risks to top management.
- Not monitoring risk adequately.
- Not managing risk adequately.
- Not using the appropriate risk metrics.

It is important for the firm to recognize all relevant risks and to measure all known risks correctly. These risks need to be managed and monitored using the appropriate risk metrics, and the results need to be properly communicated to top management.

Risk mismeasurement can occur when risk managers do not understand the distribution of returns of a single risky position or the relationships of the distributions among different positions. Understanding the distribution of a given position means being able to identify the underlying return distribution and the probabilities associated with that particular distribution. Understanding the relationships among return distributions means being able to identify how risky positions are correlated. In both cases, it is crucial to understand the degree to which return distributions and/or correlations can change over time. It is well known, for example, that correlations tend to increase during times of stress.

One of the key issues for risk managers is the occurrence of extreme events (those events which occur with low frequency, but high severity). Estimates of these rare events require a degree of subjectivity, which clearly has the potential for mismeasurement. Unfortunately, firm politics can play a role in reducing the accuracy of risk estimates since some departments may wish to understate risks by using subjective measures. Mismeasurement can also occur from ignoring relevant risks as discussed in the following AIM.

AIM 10.4: Explain how a firm can fail to take known and unknown risks into account in making strategic decisions.

Failing to take known and unknown risks into account (i.e., ignoring risks) can take three forms:

1. Ignoring a risk that is known.
2. Knowing about a risk, but failing to properly incorporate it into risk models.
3. Failing to discover all risks.

A firm ignores known risks by failing to realize how various position risks can lead to a potential disaster. This was the case when Long-Term Capital Management (LTCM) failed to recognize that high-yielding Russian debt had not only default risk, but also currency risk, sovereign risk, and counterparty risk. For example, the managers of LTCM had

thought they had hedged currency risk by selling rubles forward, but the Russian banks on the other sides of the transactions failed during the 1998 Russian crisis.

Not collecting and entering data into the appropriate risk models is another potential source of disaster. In this case, the firm may make an attempt to recognize the risk. However, not obtaining proper data to measure the risk adequately will have similar consequences to ignoring risks.

One of the severe consequences of either ignoring or not adequately using data in risk models is that the firm might expand its operations in areas where risks are not being properly accounted for. For example, consider a particular trading office within a firm where the firm has made a risk allocation to the office, but then the firm ignores the data generated by this trading office and does not monitor to see if allocation adjustments are needed. Another example is blindly accepting a given assumption (i.e., AAA-rated assets are very low risk) and ignoring data that would indicate the contrary.

Another risk that is often ignored is increasing correlations during a time of crisis. Not recognizing the possibility of increasing correlations could potentially lead to large losses. Consider, for example, the correlation between credit risk and market risk for banks. In the recent credit crisis, market risk caused decreases in security values issued through securitization, and credit risk caused decreases in the utilization of securitization. The important point is that firms must use all available data to adequately measure all risks and relationships among risks.

Some risks may go completely undetected by risk managers. Clearly, the same unfavorable outcomes discussed previously would result. In some cases, however, unknown risks may not be too severe of a problem. There are ultra-extreme events (e.g., asteroid crashing to earth) where the probability is so low and the outcome is so horrific that exploring it would not be worthwhile. Also, the nature of some risks can be unknown while their consequences are known. For example, simply knowing that a given random variable follows a normal distribution may be adequate. Furthermore, as long as management realizes that not all risks will be known and makes appropriate capital allocations to account for this, then unknown risks may not be a severe problem.

PROPERLY COMMUNICATING RISKS

AIM 10.5: Explain the importance of communication in effective risk management.

The purpose of risk management is to allow senior managers of the firm to make the optimal strategic decisions to maximize firm value. Thus, risk management efforts are wasted unless the results can be effectively communicated to the appropriate decision makers. This includes timely communication that has not been distorted by intermediaries. Furthermore, the risk management process may be harmful if there is miscommunication, and the senior managers get a false sense of security from the information that is provided. The bottom line is this—it is very important to communicate the results of the risk management process effectively.

ONGOING RISK MANAGEMENT

AIM 10.6: Describe how firms can fail to correctly monitor and manage risk on an ongoing basis.

Risk managers must recognize how portfolio risk profiles can change even during the absence of trading. The properties of some securities can change for several reasons (e.g., changes in interest rates, embedded derivatives). Also, some securities can have complex relationships with market variables; for example, a security may increase in value when interest rates decline over one particular range and then decline in value as interest rates decline further outside of that range.

The pricing of subprime derivatives serves as an example of changing risk exposures. Previously, the ABX indices (i.e., asset-backed securities indices) showed no variation for AAA-rated tranches of securitization. However, during the recent financial crisis, the values declined suddenly and dramatically, and anyone who had relied on historical values of the ABX indices for allocations incurred large losses. Obviously, a key element for successful risk management is to recognize how quickly and dramatically risk characteristics can change. Thus, it is important to be able to respond quickly and have contingency plans if/when needed.

It is also important to understand that the act of monitoring and managing risk can change the nature of risk. The **Heisenberg Principle** says that increasing the certainty for one variable may introduce uncertainty for another variable. Marking to market in one firm, for example, may start a chain reaction of adjustments in other firms which changes the risk characteristics of those firms and the overall market, thus, increasing market risk.

Another cautionary note concerning monitoring and managing risk too carefully is that it could stifle a trading department's innovation. Employees should have some degree of flexibility; therefore, a firm's management may rightly decide not to, or "fail" to, monitor and manage some risks on an ongoing basis. The downside of course is that this flexibility could make it possible for risks to emerge in remote corners of the firm. This is one of the many trade-offs senior management must consider.

Firms can fail to monitor and manage risk on an ongoing basis by not having an adequate incentive structure and/or culture that promotes effective risk management. If risk is everyone's concern, then unobserved risks are less likely. In addition, if compensation is a function of risk, then employees will likely take more interest in lowering firm risk.

THE ROLE OF RISK METRICS

AIM 10.7: Explain the role of risk metrics and discuss the shortcomings of existing risk metrics.

Risk metrics aid the management process by providing managers a target to achieve (e.g., a particular VaR level). Monitoring these risk metrics allows managers to appropriately manage risk. However, risk metrics may be too narrow in scope, which can make it more difficult to achieve the overall objective of managing risk in an effort to create value.

VaR is a widely used risk metric that is narrow in scope in several ways. Usually, a firm simply reports the number of times losses exceeded VaR in a given period. Over a year, a firm may have zero daily losses greater than daily VaR, but it could end up with an annual loss in the event that most days incurred losses (without exceeding VaR). Furthermore, for a firm that exceeds its VaR for a certain number of days, the VaR approach does not indicate the size of those losses. It is well known that VaR does not capture the implications of extremely large losses that have a very low probability of occurring.

One misuse of VaR is choosing a time period (e.g., daily or weekly) that does not correspond to the liquidity of the assets in the portfolio. Using daily VaR on a portfolio where the assets cannot be effectively traded within a day is clearly not appropriate. Taking a longer term horizon to account for liquidity of the assets may not be sufficient either. The problem is that any given horizon, such as a month or a year, may have a low probability of default because the probability of a crisis in these intermediate horizons is very low. Financial institutions generally focus on firm-wide risk management at a one-year horizon and try to achieve credit ratings that imply a low probability of default for that horizon. However, without looking ahead multiple periods, the firm has little incentive to factor in a potential crisis, which would drastically change default probabilities. The firm needs a strategy to survive those unfortunate years where crises do occur, which means that focusing on only a one-year horizon will likely fall short.

VaR also assumes the distributions of losses are not correlated over time. In the recent financial crisis, huge losses on one day led to drastic falls in liquidity, which led to large losses on the following day. The fact is that a crisis can change the nature of a return distribution for a given period as well as across periods.

Another complication is that a given firm's losses can exacerbate the risk in the overall market. This is related to an earlier discussion on how the marking-to-market of one firm can lead to adjustments in other firms. The point is that a firm with large losses in a given market can influence the activity in that market. This firm can also fall victim to predatory trading. **Predatory trading** occurs when other firms in a market see that a large player in the market is in trouble and the other firms attempt to push the price down further in order to hurt the large player. Such activity is difficult to incorporate into risk metrics.

In its risk management process, a firm can attempt to capture such complications with scenario analysis. The scenarios would include a crises and/or a firm's behavior in the overall market. Scenario analysis requires input from people who have a solid understanding of not only mathematics, but also the complexities of human behavior.

KEY CONCEPTS

1. Risk management involves assessing, communicating, monitoring, and managing risks.
2. A large loss does not necessarily mean that risk management has failed. Losses are the result of risk taking, which is required for value creation.
3. Risk management can fail if the firm does not do the following: measure risks correctly, recognize some risk, communicate risks to top management, monitor and manage risks, and use appropriate metrics.
4. Mismeasurement can occur when management does not understand the distribution of returns of a single position or the relationships of the distributions among positions and how the distributions and correlations can change over time.
5. Mismeasurement can also occur when managers must use subjective probabilities for rare and extreme events. The subjective probabilities can be biased from firm politics.
6. Failing to take known and unknown risks into account can take three forms: (1) ignore a risk that is known, (2) failure to incorporate a risk into risk models, and (3) not finding all risks. All three of these are variations of the same concept and can have similar results (e.g., failure to measure overall risk or expanding operations to areas where risk is not being properly measured).
7. Senior managers must understand the results of risk management in order for it to be meaningful. Unless senior managers have the correct information to make decisions, risk management is pointless.
8. Risk managers must recognize how risk characteristics change over time. Many securities have complex relationships with market variables.
9. Having an adequate incentive structure and firm-wide culture can help with the risk monitoring and managing process.
10. Risk metrics such as VaR are usually too narrow in scope. For example, VaR usually assumes independent losses across periods of time.
11. Risk metrics generally fail to capture the effect of a firm's actions on the overall market and behavior patterns such as predatory trading.

CONCEPT CHECKERS

1. Which of the following is not part of the risk management process?
 - A. Monitoring risk.
 - B. Assessing the risks faced by the firm.
 - C. Properly communicating the risks to upper management.
 - D. Reducing the probability of loss to as close to zero as possible.
2. Paul Frank, FRM, manages several positions within a portfolio. He has determined all possible outcomes for every single position. The result of his detailed work means that:
 - A. risk mismeasurement is not possible.
 - B. risk mismeasurement is still possible for each of the positions and the overall portfolio.
 - C. risk mismeasurement is only possible if the possible outcomes change and Frank does not make the necessary adjustments.
 - D. risk mismeasurement is not possible for each of the positions, but it is possible for the overall portfolio because correlations have not been addressed.
3. The Tower Corporation has several divisions, and each must give updated reports on its risk levels. The nature of Tower's business is that there is the possibility of large losses that are very infrequent, some of which have never actually been realized. Tower requires that the manager of each division include subjective assessments of these risks in their reports. With respect to this risk assessment, which of the following statements is most accurate? This action:
 - A. is the best way to avoid risk mismeasurement.
 - B. is always an appropriate method for managing risk.
 - C. can be a source of risk mismeasurement, but Tower can expect the errors to be unbiased.
 - D. can be a source of risk mismeasurement due to the subjective input and the fact that there may be bias in the input.
4. If risk managers are not certain of all risks faced by the firm:
 - A. the firm will most likely fail.
 - B. this can be a source of risk management failure, but not in all cases.
 - C. this is a cause of risk management failure and is always avoided with adequate research.
 - D. this is a source of risk management failure and usually cannot be avoided with adequate research.
5. Crane Corporation has a multi-tier management structure. Risk management occurs in each division at the base level of the structure (i.e., in each division). The results of the process are then successfully communicated to higher tiers, where it is reviewed and revised at each tier, and then sent to the appropriate decision makers for the firm. This process is:
 - A. not appropriate because it allows for distortions between the managers of risk and the decision makers.
 - B. not appropriate because risk management should not be done at the base level of a corporation.
 - C. appropriate, and it would also be appropriate to have the base tier report directly to the top management tier.
 - D. appropriate because it allows for maximum input into the process.

CONCEPT CHECKER ANSWERS

1. D Some losses are to be expected if risk taking is aimed at creating value.
2. B Frank must also consider the probabilities of the outcomes and not just the outcomes themselves. He must also consider the correlations across positions.
3. D Subjective inputs will have random errors and, in this case, may very well exhibit bias because each manager likely has a motive to understate risk.
4. B Some risks may not be known explicitly, but they can still be accounted for. In this case, risk management can still be successful. Also, not knowing the risks themselves but understanding the results of the risk (i.e., the distribution of returns) can be adequate for successful risk management.
5. A The fact that intermediate tiers can modify the information without being directly involved in the risk management process can introduce distortions.

The following is a review of the Foundations of Risk Management principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

GARP CODE OF CONDUCT

Topic 11

EXAM FOCUS

This topic addresses the GARP Code of Conduct which sets forth principles related to ethical behavior within the risk management profession. FRM candidates are expected to know all Member responsibilities as well as sanctions that could result if violations of the Code occur. The material in this topic is relatively easy to understand; however, for the exam, you should expect complex questions related to these ethical standards that test whether or not a violation has occurred.

The GARP Code of Conduct contains a set of key principles designed to support financial risk management practices. The Code was developed for the Financial Risk Manager (FRM) program as well as other certification programs administered by the Global Association of Risk Professionals (GARP). All GARP Members (including FRM candidates) are expected to abide by the principles outlined in the Code and are subject to consequences, such as suspensions, for violating any parts of the Code.

A GARP Member should understand that high ethical behavior goes beyond the principles addressed in this topic. When encountering a situation not specifically outlined in the Code, Members are always expected to act in an ethical fashion. Acting with prudence in all situations related to the profession will uphold the integrity of the risk management field as well as risk management practitioners.

THE CODE OF CONDUCT¹

The Code of Conduct stresses ethical behavior in two areas: (1) Principles and (2) Professional Standards. The Principles section addresses: (1) professional integrity and ethical conduct, (2) conflicts of interest, and (3) confidentiality. The Professional Standards section addresses: (1) fundamental responsibilities and (2) adherence to generally accepted practices in risk management. The responsibilities listed in each section are examined in the following AIM.

1. Copyright 2010, Global Association of Risk Professionals. Reproduced and republished from "Code of Conduct" with permission from GARP. All rights reserved. Retrieved December 1, 2011, from <http://www.garp.org/media/59589/code%20of%20conduct0610.pdf>.

AIM 11.1: Describe the responsibility of each GARP Member with respect to professional integrity, ethical conduct, conflicts of interest, confidentiality of information and adherence to generally accepted practices in risk management.

1. Professional Integrity and Ethical Conduct

GARP Members:

- 1.1. shall act professionally, ethically and with integrity in all dealings with employers, existing or potential clients, the public, and other practitioners in the financial services industry.
- 1.2. shall exercise reasonable judgment in the provision of risk services while maintaining independence of thought and direction. GARP Members must not offer, solicit, or accept any gift, benefit, compensation, or consideration that could be reasonably expected to compromise their own or another's independence and objectivity.
- 1.3. must take reasonable precautions to ensure that the Member's services are not used for improper, fraudulent or illegal purposes.
- 1.4. shall not knowingly misrepresent details relating to analysis, recommendations, actions, or other professional activities.
- 1.5. shall not engage in any professional conduct involving dishonesty or deception or engage in any act that reflects negatively on their integrity, character, trustworthiness, or professional ability or on the risk management profession.
- 1.6. shall not engage in any conduct or commit any act that compromises the integrity of GARP, the FRM® designation, or the integrity or validity of the examinations leading to the award of the right to use the FRM designation or any other credentials that may be offered by GARP.
- 1.7. shall be mindful of cultural differences regarding ethical behavior and customs, and avoid any actions that are, or may have the appearance of being unethical according to local customs. If there appears to be a conflict or overlap of standards, the GARP Member should always seek to apply the highest standard.

2. Conflict of Interest

GARP Members shall:

- 2.1. act fairly in all situations and must fully disclose any actual or potential conflict to all affected parties.
- 2.2. make full and fair disclosure of all matters that could reasonably be expected to impair independence and objectivity or interfere with respective duties to their employer, clients, and prospective clients.

3. Confidentiality

GARP Members:

- 3.1. shall not make use of confidential information for inappropriate purposes and unless having received prior consent shall maintain the confidentiality of their work, their employer or client.
- 3.2. must not use confidential information for personal benefit.

4. Fundamental Responsibilities

GARP Members shall:

- 4.1. comply with all applicable laws, rules, and regulations (including this Code) governing the GARP Members' professional activities and shall not knowingly participate or assist in any violation of such laws, rules, or regulations.
- 4.2. have ethical responsibilities and cannot outsource or delegate those responsibilities to others.
- 4.3. understand the needs and complexity of their employer or client, and should provide appropriate and suitable risk management services and advice.
- 4.4. be diligent about not overstating the accuracy or certainty of results or conclusions.
- 4.5. clearly disclose the relevant limits of their specific knowledge and expertise concerning risk assessment, industry practices, and applicable laws and regulations.

5. Best Practices

GARP Members shall:

- 5.1. execute all services with diligence and perform all work in a manner that is independent from interested parties. GARP Members should collect, analyze and distribute risk information with the highest level of professional objectivity.
- 5.2. be familiar with current generally accepted risk management practices and shall clearly indicate any departure from their use.
- 5.3. ensure that communications include factual data and do not contain false information.
- 5.4. make a distinction between fact and opinion in the presentation of analysis and recommendations.

VIOLATIONS OF THE CODE OF CONDUCT

AIM 11.2: Describe the potential consequences of violating the GARP Code of Conduct.

All GARP Members are expected to act in accordance with the GARP Code of Conduct as well as any local laws and regulations that pertain to the risk management profession. If the Code and certain laws conflict, then laws and regulations will take priority.

Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership. In addition, violations could lead to a revocation of the right to use the FRM designation. Sanctions would be issued after a formal investigation is conducted by GARP.



Professor's Note: There are no Key Concepts for this short topic.

CONCEPT CHECKERS

1. Over the past two days, Lorraine Quigley, FRM, manager of a hedge fund, has been purchasing large quantities of Craeger Industrial Products' common stock while at the same time shorting put options on the same stock. Quigley did not notify her clients of the trades although they are aware of the fund's general strategy to generate returns. Which of the following statements is most likely correct? Quigley:
 - A. did not violate the Code.
 - B. violated the Code by manipulating the prices of publicly traded securities.
 - C. violated the Code by failing to disclose the transactions to clients before they occurred.
 - D. violated the Code by failing to establish a reasonable and adequate basis before making the trades.

2. Jack Schleifer, FRM, is an analyst for Brown Investment Managers (BIM). Schleifer has recently accepted an invitation to visit the facilities of ChemCo, a producer of chemical compounds used in a variety of industries. ChemCo offers to pay for Schleifer's accommodations in a penthouse suite at a luxury hotel and allow Schleifer to use the firm's private jet to travel to its three facilities located in New York, Hong Kong, and London. In addition, ChemCo offers two tickets to a formal high-society dinner in New York. Schleifer declines to use ChemCo's corporate jet or to allow the firm to pay for his accommodations but accepts the tickets to the dinner (which he discloses to his employer) since he will be able to market his firm's mutual funds to other guests at the dinner. Has Schleifer violated the GARP Code of Conduct?
 - A. Yes.
 - B. No, since he is using the gifts accepted to benefit his employer's interests.
 - C. No, since the gifts he accepted were fully disclosed in writing to his employer.
 - D. No, since the gift he accepted is of nominal value and he declined to accept the hotel accommodations and the use of ChemCo's jet.

3. Beth Bixby, FRM, oversees a mid-cap fund that is required to invest in a minimum of 40 and a maximum of 60 different issues. Bixby uses a quantitative approach to actively manage the assets. In promotional materials, she states that "through our complex quantitative approach, securities are selected that have similar exposures to a number of risk factors that are found in the S&P 500 Index. Thus the fund is designed to track the performance of the S&P 500 Index but will receive a return premium of between 2% and 4% according to our model's risk-return measures." This statement is:
 - A. permissible since the assertion is supported by modern portfolio theory and estimates from the firms' model.
 - B. not permissible since Bixby is misrepresenting the services that she and/or her firm are capable of performing.
 - C. not permissible since Bixby is misrepresenting the investment performance she and/or her firm can reasonably expect to achieve.
 - D. permissible since the statement describes the basic characteristics of the fund's risk and return objectives.

4. Gail Stefano, FRM, an analyst for a U.S. brokerage firm that serves U.S. investors, researches public utilities in South American emerging markets. Stefano makes the following statement in a recent report: “Based on the fact that the South American utilities sector has seen rapid growth in new service orders, we expect that most companies in the sector will be able to convert the revenue increases into significant profits. We also believe the trend will continue for the next three to five years.” The report goes on to describe the major risks of investing in this market, in particular the political and exchange rate instability associated with South American countries. Stefano’s report:
 - A. has not violated the Code.
 - B. violated the Code by failing to properly distinguish factual information from opinions.
 - C. violated the Code by recommending an investment which would not be suitable for all of its clients.
 - D. violated the Code by failing to properly identify details related to the operations of South American utilities.

5. Beth Anderson, FRM, is a portfolio manager for several wealthy clients including Reuben Carlyle. Anderson manages Carlyle’s personal portfolio of stock and bond investments. Carlyle recently told Anderson that he is under investigation by the IRS for tax evasion related to his business, Carlyle Concrete (CC). After learning about the investigation, Anderson proceeds to inform a friend at a local investment bank so that they may withdraw their proposal to take CC public. Which of the following is most likely correct? Anderson:
 - A. violated the Code by failing to immediately terminate the client relationship with Carlyle.
 - B. violated the Code by failing to maintain the confidentiality of her client’s information.
 - C. violated the Code by failing to detect and report the tax evasion to the proper authorities.
 - D. did not violate the Code since the information she conveyed pertained to illegal activities on the part of her client.

CONCEPT CHECKER ANSWERS

1. A Quigley's trades are most likely an attempt to take advantage of an arbitrage opportunity that exists between Craeger's common stock and its put options. She is not manipulating the prices of securities in an attempt to mislead market participants. She is pursuing a legitimate investment strategy. Participants in her hedge fund are aware of the fund's investment strategy, and thus Quigley did not violate the Code by not disclosing this specific set of trades in advance of trading (Standards 2.1 and 5.1).
2. A GARP Members must not offer, solicit, or accept any gift, benefit, compensation, or consideration that could be reasonably expected to compromise their own or another's independence and objectivity. Schleifer has appropriately rejected the offer of the hotel accommodations and the use of ChemCo's jet. However, Schleifer cannot accept the tickets to the dinner. Since it is a formal high-society dinner, the tickets are most likely expensive or hard to come by. Even though he has disclosed the gift to his employer and he plans to use the dinner as a marketing opportunity for his firm, the gift itself may influence Schleifer's future research in favor of ChemCo. Allowing such potential influence is a violation of Professional Integrity and Ethical Conduct (Standard 1.2).
3. C It is not reasonable for Bixby to expect a 40-to-60 stock mid-cap portfolio to track the entire S&P 500 Index, which is a large-cap index. She should know that there will be periods of wide variance between the performance of the portfolio and the S&P 500 Index. There is no assurance that a premium of 2% to 4% will consistently be obtained. Bixby is in violation of Standard 1.4: "GARP Members shall not knowingly misrepresent details relating to analysis, recommendations, actions, or other professional activities," since she has made an implicit guarantee of the fund's expected performance.
4. A Historical growth can be cited as a fact since it actually happened. Stefano states that her firm expects further growth and profitability which is an opinion. She does not claim that these are facts. Thus, she is not in violation of Standard 5.4. In addition, Stefano identifies relevant factors and highlights in particular the most significant risks of investing in South American utilities. She has fully complied with Standard 5.3.
5. B Anderson must maintain the confidentiality of client information according to Standard 3.1. Confidentiality may be broken in instances involving illegal activities on the part of the client, but the client's information may only be relayed to proper authorities. Anderson did not have the right to inform the investment bank of her client's investigation.

CHALLENGE PROBLEMS

Use the following data to answer Questions 1 through 5.

Samuel Perkins invests his clients' assets in combinations of the risk-free asset and the market portfolio. Current market expectations are as follows:

- Expected return on the market portfolio 12%
- Standard deviation on the market portfolio 20%
- Risk-free rate 4%

1. Perkins advises a client who would like to have a portfolio with a standard deviation equal to 10%. Using the market portfolio and risk-free asset, a portfolio meeting the client's risk tolerance will have an expected return closest to:
A. 6%.
B. 8%.
C. 10%.
D. 12%.
2. Perkins advises another client who currently owns a portfolio with an expected return of 8% and a standard deviation of 15%. The amount (percentage points) by which Perkins can improve his client's expected return by using the risk-free asset and the market portfolio while maintaining the client's 15% standard deviation is closest to:
A. 2%.
B. 4%.
C. 6%.
D. 8%.
3. One of Perkins's clients has an expected return objective of 10%. Using the risk-free asset and the market portfolio, Perkins can create a portfolio with a standard deviation as low as:
A. 13%.
B. 14%.
C. 15%.
D. 16%.
4. What is the appropriate allocation to the optimal risky portfolio for a client who has a 10% standard deviation objective?
A. 30%.
B. 40%.
C. 50%.
D. 60%.

5. From the data provided, the intercept and slope of the efficient frontier constructed from the market portfolio and the risk-free asset are closest to:
- A. 4% intercept and an 8% slope.
 - B. 4% intercept and a 40% slope.
 - C. 8% intercept and an 8% slope.
 - D. 8% intercept and a 40% slope.
6. Over a decade ago, Yasuo Hamanaka, the lead copper trader for Sumitomo, attempted to corner the copper market in a classic market manipulation strategy. Such lack of supervision over his trading activities resulted from poor internal controls. Because of that lack of supervision, which of the following series of transactions was he able to engage in that ultimately resulted in a \$2.6 billion trading loss for Sumitomo?
- A. Long physical copper, short futures contracts, bought put options.
 - B. Short physical copper, long futures contracts, sold put options.
 - C. Long physical copper, long futures contracts, sold put options.
 - D. Short physical copper, short futures contracts, bought put options.

CHALLENGE PROBLEM ANSWERS

1. B The equation for the CML is:

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_C, \text{ where } R_F = 0.04$$

$$\left[\frac{E(R_M) - R_F}{\sigma_M} \right] = \left[\frac{0.12 - 0.04}{0.20} \right] = 0.40$$

Therefore, the equation of the CML is:

$$E(R_C) = 0.04 + 0.40\sigma_C$$

Setting the standard deviation equal to 0.10:

$$E(R_C) = 0.04 + 0.40(0.10) = 0.08 = 8\%$$

(See Topic 2)

2. A The equation for the CML is: $E(R_C) = 0.04 + 0.40\sigma_C$. Setting the standard deviation equal to 0.15:

$$E(R_C) = 0.04 + 0.40(0.15) = 0.10 = 10\%$$

Therefore, Perkins can improve the client's expected return by two percentage points:

new expected return = 10%

old expected return = 8%

(See Topic 2)

3. C The equation for the CML is: $E(R_C) = 0.04 + 0.40\sigma_C$. Setting the expected return to 0.10:

$$E(R_C) = 0.10 = 0.04 + 0.40\sigma_C$$

Solving for σ_C :

$$0.40\sigma_C = 0.10 - 0.04 = 0.06$$

$$\sigma_C = 0.06/0.40 = 0.15 = 15\%$$

(See Topic 2)

4. C The standard deviation for the investment combination (of treasury bills and the market portfolio) equals:

$$\sigma_C = w_M \sigma_M$$

The client wants $\sigma_C = 0.10$.

Therefore, $0.10 = w_M(0.20)$.

$$w_M = 0.10/0.20 = 0.50 = 50\%$$

Therefore, the portfolio should be allocated 50% to Treasury bills and 50% to the market portfolio.

(See Topic 2)

5. B The equation for the CML is:

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_C$$

where the intercept is the risk-free rate, $R_F = 0.04$, or 4%, and the slope equals the market risk premium $[E(R_M) - R_F]$ per unit of market risk, σ_M .

$$\left[\frac{E(R_M) - R_F}{\sigma_M} \right] = \left[\frac{0.12 - 0.04}{0.20} \right] = 0.40, \text{ or } 40\%$$

(See Topic 2)

6. C Hamanaka established a dominant long position in futures contracts and simultaneously purchased large quantities of physical copper. As well, to help finance his long copper positions, he even sold put options on copper. In essence, here was a “triple long” strategy that would only pay off if the price of copper or copper futures increased. At the same time, there was a huge risk of losses should the prices fall. Unfortunately, there was a continuation of plummeting copper prices after other copper traders began selling their copper holdings in anticipation of Sumitomo doing the same. The end result was total losses of \$2.6 billion for Sumitomo.

(See Topic 9)

GARP FRM PRACTICE EXAM QUESTIONS

Foundations of Risk Management



Professor's Note: The following questions are from the 2008–2011 GARP FRM Practice Exams.

1. John Diamond is evaluating the existing risk management system of Rome Asset Management and identified the following two risks.
 - I. Rome Asset Management's derivative pricing model consistently undervalues call options
 - II. Swaps with counterparties exceed counterparty credit limit

These two risks are most likely to be classified as:

- A. Market
 - B. Credit
 - C. Liquidity
 - D. Operational
-
2. If the daily, 90% confidence level, value at risk (VaR) of a portfolio is correctly estimated to be USD 5,000, one would expect that in one out of:
 - A. 10 days, the portfolio value will decline by USD 5,000 or less.
 - B. 90 days, the portfolio value will decline by USD 5,000 or less.
 - C. 10 days, the portfolio value will decline by USD 5,000 or more.
 - D. 90 days, the portfolio value will decline by USD 5,000 or more.

 3. John Diamond is evaluating the existing risk management system of Rome Asset Management and identified the following two risks.
 - I. Credit spreads widen following recent bankruptcies
 - II. The bid-ask spread of an asset suddenly widens

Which of these can be identified as liquidity risk?

- A. I only
- B. II only
- C. I and II
- D. Neither

4. A global investment risk manager is assessing an investment's performance using a two-factor model. In order to determine the volatility of the investment, the risk manager developed the following factor covariance matrix for global assets:

Factor Covariance Matrix for Global Assets

	<i>Global Equity Factor</i>	<i>Global Bond Factor</i>
Global Equity Factor	0.24500	0.00791
Global Bond Factor	0.00791	0.01250

Suppose the factor sensitivity to the global equity factor is 0.75 for the investment and the factor sensitivity to the global bond factor is 0.20 for the investment. The volatility of the investment is closest to:

- A. 11.5%
B. 24.2%
C. 37.5%
D. 42.2%
5. Tim is evaluating 4 funds run by 4 independent managers relative to a benchmark portfolio that has an expected return of 7.4% and volatility of 14%. He is interested in investing in the fund with the highest information ratio that also meets the following conditions in his investment guidelines:
- Expected residual return must be at least 2%
 - Residual risk relative to the benchmark portfolio must be less than 2.5%

Based on the following information, which fund should he choose?

<i>Fund</i>	<i>Expected Return</i>	<i>Volatility</i>	<i>Residual Risk</i>	<i>Information Ratio</i>
Fund A	9.3%	15.3%		0.8
Fund B		16.4%	2.4%	0.9
Fund C		15.8%	1.5%	1.3
Fund D	9.4%		1.8%	

- A. Fund A
B. Fund B
C. Fund C
D. Fund D

6. Tom is evaluating 4 funds run by 4 independent managers relative to a benchmark portfolio that has an expected return of 6.4% and volatility of 12%. He is interested in investing in the fund with the highest information ratio that also meets the following conditions in his investment guidelines:

- I. Expected residual return must be at least 2%
- II. The Sharpe ratio must be at least 0.2

Based on the following information, which fund should he choose?

<i>Fund</i>	<i>Expected Return</i>	<i>Volatility</i>	<i>Residual Risk</i>	<i>Information Ratio</i>
Fund A	8.4%	14.3%		1.1
Fund B		16.4%	2.4%	0.9
Fund C		17.8%	1.5%	1.3
Fund D	8.5%	19.1%	1.8%	

- A. Fund A
 - B. Fund B
 - C. Fund C
 - D. Fund D
7. Brisk Holdings, a large conglomerate, is implementing the enterprise risk management (ERM) framework to quantify and manage the risk-return tradeoff for the entire firm. Which of the following statements about the ERM framework is/are correct?
- I. The performance of each business unit should be evaluated on a stand-alone basis and the unit should be allocated more capital if its net income is positive.
 - II. The ERM framework tries to minimize the aggregate risk taken by the firm.
- A. Statement I only
 - B. Statement II only
 - C. Both statements are correct
 - D. Both statements are incorrect
8. Jennifer Durrant is evaluating the existing risk management system of Silverman Asset Management. She is asked to match the following events to the corresponding type of risk. Identify each numbered event as a market risk, credit risk, operational risk, or legal risk event.
- Event**
- 1. Insufficient training leads to misuse of order management system.
 - 2. Credit spreads widen following recent bankruptcies.
 - 3. Option writer does not have the resources required to honor a contract.
 - 4. Credit swaps with counterparty cannot be netted because they originated in multiple jurisdictions.
- A. 1: legal risk, 2: credit risk, 3: operational risk, 4: credit risk
 - B. 1: operational risk, 2: credit risk, 3: operational risk, 4: legal risk
 - C. 1: operational risk, 2: market risk, 3: credit risk, 4: legal risk
 - D. 1: operational risk, 2: market risk, 3: operational risk, 4: legal risk

9. Portfolio Q has a beta of 0.7 and an expected return of 12.8%. The market risk premium is 5.25%. The risk-free rate is 4.85%. Calculate Jensen's Alpha measure for Portfolio Q.
- 7.67%
 - 2.70%
 - 5.73%
 - 4.27%
10. A corporation is faced with the decision to choose between the two following projects:

<i>Project</i>	<i>Investment</i>	<i>Perpetual Annual Cash Flow</i>	<i>Cash Flow at Risk</i>
A	100	20	50
B	80	55	200

- Assuming that there is no systematic risk and the projects are mutually exclusive, under what circumstances would project A be selected over project B?
- Project A should never be chosen because it requires a larger initial investment and generates lower perpetual annual cash flows.
 - Project A could be preferred over Project B if Project A's cash flows are negatively correlated with the firm's existing cash flows while the cash flows of Project B are highly positively correlated with the firm's existing cash flows.
 - Project A should be chosen if the opportunity cost of funds is low, and Project B should be chosen otherwise.
 - Project A should be chosen if the net present value of the project is positive.
11. There are many reasons why risk management increases shareholder wealth. Which of the following risk management policies is least likely to increase shareholder wealth?
- Hedging strategies to lower the probability of financial distress and bankruptcy.
 - Risk management policies designed to reduce the probability of debt overhang.
 - Well-designed compensation structure for managers that sets incentives for managers to take appropriate risks.
 - Risk management policies designed to eliminate projects with high volatility.
12. In late 1993, Metallgesellschaft reported losses of approximately USD 1.5 billion in connection with the implementation of a hedging strategy in the oil futures market. In 1992, the company had begun a new strategy to sell petroleum to independent retailers, on a monthly basis, at fixed prices above the prevailing market price for periods of up to 5 and even 10 years. At the same time, Metallgesellschaft implemented a hedging strategy using a large number of short-term derivative contracts such as swaps and futures on crude oil, heating oil, and gasoline on several exchanges and markets. Its approach was to buy on the derivatives market exposure to one barrel of oil for each barrel it had committed to deliver. Because of its choice of a hedge ratio, the company suffered significant losses with its hedging strategy when oil market conditions abruptly changed to:
- Contango, which occurs when the futures price is above the spot price.
 - Contango, which occurs when the futures price is below the spot price.
 - Normal backwardation, which occurs when the futures price is above the spot price.
 - Normal backwardation, which occurs when the futures price is below the spot price.

13. The information ratio of the Sterole US Fund for 2006 against the S&P 500, its benchmark index, is 1. For the same time period, the fund's Sharpe ratio is 2, the fund has a tracking error of 7% against the S&P 500, and the standard deviation of fund returns is 5%. The risk-free rate in the US is 4%. Calculate the return for the S&P 500 during the time period.
- A. 3.5%
 - B. 7%
 - C. 11%
 - D. 14%
14. A fund manager recently received a report on the performance of his portfolio over the last year. According to the report, the portfolio return is 9.3%, with a standard deviation of 13.5%, and a beta of 0.83. The risk-free rate is 3.2%, the semi-standard deviation $\sigma_L(R_p)$ of the portfolio is 8.4%, and the tracking error of the portfolio to the benchmark index is 2.8%. What is the difference between the value of the fund's Sortino ratio (computed relative to the risk-free rate) and its Sharpe ratio?
- A. 0.274
 - B. 1.727
 - C. 0.653
 - D. -0.378
15. Which of the following risk management strategies of a firm which has principal payments to make on its debt in one year that substantially exceed the market value of its assets is most likely to be in the interest of the shareholders?
- A. Reduction of the overall risk level of the firm
 - B. Increase of the overall risk level of the firm
 - C. Keep the same risk level
 - D. It is impossible to say which risk management strategy the shareholders prefer
16. An analyst has compiled the following information on a portfolio:
- Sortino Ratio: 0.82
 - Beta: 1.15
 - Expected return: 12.2%
 - Standard deviation: 16.4%
 - Benchmark return: 11.9%
 - Risk-free rate: 4.75%
- Calculate the semi-standard deviation of the portfolio.
- A. 0.4%
 - B. 8.2%
 - C. 14.9%
 - D. 9.08%

17. Suppose the daily returns of a portfolio and a benchmark portfolio it is replicating are as follows:

	<i>Portfolio Return (bps)</i>	<i>Benchmark Portfolio Return (bps)</i>
Day 1	34	30
Day 2	-89	-87
Day 3	108	102
Day 4	70	70

What is the tracking error over the four day period?

- A. 3.16 bps
B. 2 bps
C. 10 bps
D. 2.39 bps
18. Assume that a portfolio underperformed its benchmark by 2% in the most recent month. In this scenario:
A. Alpha is “-2%” as it refers to the Outperformance / Underperformance Gap.
B. due to underperformance, Alpha is definitely negative and cannot be positive.
C. Alpha may be positive or negative depending upon Beta and Risk Free Rate.
D. Alpha is 2%.
19. Which of the following statements about the Sortino ratio are valid?
I. The Sortino ratio is more appropriate for asymmetrical return distributions.
II. The Sortino ratio compares the portfolio return to the return of a benchmark portfolio.
III. The Sortino ratio allows one to evaluate portfolios obtained through an optimization algorithm that uses variance as a risk metric.
IV. The Sortino ratio is defined on the same principles as the Sharpe ratio, but the Sortino ratio replaces the risk free rate with the minimum acceptable return and the standard deviation of returns with the standard deviation of returns below the minimum acceptable return.
A. II and III
B. I, III and IV
C. I and III
D. I and IV
20. A portfolio has an average return over the last year of 13.2%. Its benchmark has provided an average return over the same period of 12.3%. The portfolio’s standard deviation is 15.3%, its beta is 1.15, its tracking error volatility is 6.5% and its semi-standard deviation is 9.4%. Lastly the risk free rate is 4.5%.
Calculate the portfolio’s Information Ratio (IR).
A. 0.569
B. 0.076
C. 0.138
D. 0.096

21. Which of the following statements is not correct?
- A. The more the firm hedges its financial exposures, the less equity it requires to support its business.
 - B. In order to maximize the value, a firm must hedge its financial exposure irrespective of its capital structure.
 - C. The use of risk management to reduce financial exposures effectively increases a firm's debt capacity.
 - D. Decisions to hedge financial exposures should be made jointly with the company's capital structure decisions.
22. Which of the following cases of losses was not the result of unauthorized or rogue trading?
- A. Long-Term Capital Management
 - B. Allied Irish Bank
 - C. Sumitomo
 - D. Daiwa

GARP FRM PRACTICE EXAM ANSWERS

Foundations of Risk Management

Question from the 2011 FRM practice exam.

1. D I is a model failure and II is an internal failure. These are types of operational risks.
(See Topic 1)

Question from the 2011 FRM practice exam.

2. C If the daily, 90% confidence level value at risk (VaR) of a portfolio is correctly estimated to be USD 5,000, one would expect that 90% of the time (9 out of 10), the portfolio will lose less than USD 5,000; equivalently, 10% of the time (1 out of 10) the portfolio will lose USD 5,000 or more.
(See Topic 1)

Question from the 2011 FRM practice exam.

3. B I is market risk, II is liquidity risk.
(See Topic 1)

Question from the 2011 FRM practice exam.

4. C
$$\begin{aligned}\text{Var}(\text{Inv}) &= \beta_1^2 \sigma_{F_1}^2 + \beta_2^2 \sigma_{F_2}^2 + 2\beta_1\beta_2 \text{Cov}(F_1, F_2) \\ &= (0.75)^2 (0.245) + (0.20)^2 (0.0125) + 2(0.75)(0.20)(0.00791) \\ &= 0.1407\end{aligned}$$

$$\sigma = \sqrt{0.1407} = 37.5\%$$

(See Topics 2 and 5)

Question from the 2011 FRM practice exam.

5. D Information ratio = expected residual return / residual risk = $E(R_P - R_B) / \sigma(R_P - R_B)$

Fund A: Expected residual return = $9.3\% - 7.4\% = 1.9\%$, which does not meet the requirement of minimum residual return of 2%.

Fund B: Expected residual return = information ratio * residual risk = $0.9 * 2.4\% = 2.16\%$, so it meets both requirements.

Fund C: Expected residual return = information ratio * residual risk = $1.3 * 1.5\% = 1.95\%$, does not meet residual return of 2%.

Fund D: This fund also meets both the residual return and residual risk requirements.

$$\text{Expected residual return} = 9.4\% - 7.4\% = 2.0\%$$

$$\text{Information ratio} = 2.0\% / 1.8\% = 1.11$$

Both funds B and D meet the requirements. Fund D has the higher information ratio.

(See Topic 6)

Question from the 2011 FRM practice exam.

6. A Sharpe Ratio = Return Premium over Risk Free Rate / Volatility = $E(R_p - R_f) / \sigma$

Fund A: Expected residual return = $8.4\% - 6.4\% = 2.0\%$

Sharpe Ratio = $(8.4\% - 5\%) / 14.3\% = 0.238$

Fund B: Expected residual return = information ratio * residual risk = $0.9 * 2.4\% = 2.16\%$

Sharpe Ratio = $(2.16\% + 6.4\% - 5\%) / 16.4\% = 0.217$

Fund C: Expected residual return = information ratio * residual risk = $1.3 * 1.5\% = 1.95\%$

Fund D: Expected residual return = $8.5\% - 6.4\% = 2.1\%$

Information ratio = $2.1\% / 1.8\% = 1.16$

Sharpe Ratio = $(8.5\% - 5\%) / 19.1\% = 0.183$

Both funds A and B meet the requirements. Fund A has the higher information ratio.

(See Topic 6)

Question from the 2011 FRM practice exam.

7. D Statement I is incorrect. Management must avoid a silo approach in its evaluation of the performance of each business unit but should take into account the contributions of each of the units to the firm's total risk. This can be done by assigning a level of additional imputed capital to reflect incremental risk of the project.

Statement II is incorrect. The purpose of an ERM program is not to minimize or eliminate the firm's probability of distress. Rather, it should optimize the firm's risk portfolio by trading off the probability of large shortfalls and its associated costs and with expected gains from taking strategic and business risks.

(See Topic 7)

Question from the 2010 FRM practice exam.

8. C A, B and D are incorrect. C is correct.

1. Insufficient training leads to misuse of order management system is an example of operational risk.

2. Widening of credit spreads represents an increase in market risk.

3. An option writer not honoring the obligation in a contract is a credit risk event.

4. When a contract is originated in multiple jurisdictions leading to problems with enforceability, there is legal risk.

(See Topic 1)

Question from the 2010 FRM practice exam.

9. D Jensen's alpha is defined by:

$$E(R_P) - R_F = \alpha_P + \beta_P(E(R_M) - R_F); \alpha_P = E(R_P) - R_F - \beta_P(E(R_M) - R_F) = 0.128 - 0.0485 - 0.7 * (0.0525 + 0.0485 - 0.0485) = 0.0427$$

- A. Incorrect. Forgets to subtract the risk-free rate for the excess market return.
- B. Incorrect. Forgets to multiply the excess market return by beta.
- C. Incorrect. Forgets to subtract the risk-free rate for both the excess market return and the excess portfolio return.
- D. Correct.

(See Topic 6)

Question from the 2010 FRM practice exam.

10. B Project A should be chosen only if the cash flow at risk of the project has low or negative correlation with the other projects the company currently has or plans. The overall cash flow position of the firm has to be evaluated as a result.

(See Topic 8)

Question from the 2010 FRM practice exam.

11. D The first three are examples of where risk management can increase firm value. The last one is invalid because reducing volatility per se could just eliminate projects with extremely high payoffs.

(See Topic 8)

Question from the 2010 FRM practice exam.

12. A Oil prices fell in the fall of 1993 because of OPEC's problems adhering to its production quotas, so the market changed into one of contango so C and D are incorrect. In contango, the futures price is above the spot price and as a result Metallgesellschaft incurred losses on its short-dated long futures contracts so B is incorrect and A is correct.

(See Topic 9)

Question from the 2009 FRM practice exam.

13. B Sharpe Ratio = 2
 $(\text{Fund Return} - \text{Risk Free Rate}) / \text{SD} = 2$
 $(\text{Fund Return} - 4\%) / 5\% = 2$
Fund Return = 14%
- Information Ratio = 1
 $(\text{Fund Return} - \text{S\&P 500 Return}) / \text{Tracking Error} = 1$
 $(14\% - \text{S\&P 500 Return}) / 7\% = 1$
S\&P 500 Return = 7%
- A. Incorrect. Incorrectly divides S\&P 500 return by 2.
C. Incorrect. The candidate might use the tracking error as the denominator in both the ratios.
Sharpe Ratio = 2
 $(\text{Fund Return} - \text{Risk Free Rate}) / \text{Tracking Error} = 2$
 $(\text{Fund Return} - 4\%) / 7\% = 2$
Fund Return = 18%
Information Ratio = 1
 $(\text{Fund Return} - \text{S\&P 500 Return}) / \text{Tracking Error} = 1$
 $(18\% - \text{S\&P 500 Return}) / 7\% = 1$
S\&P 500 Return = 11%
D. Incorrect. The candidate can stop with the fund return calculation, and end up with 14%.
Sharpe Ratio = 2
 $(\text{Fund Return} - \text{Risk Free Rate}) / \text{SD} = 2$
 $(\text{Fund Return} - 4\%) / 5\% = 2$
Fund Return = 14%
(See Topic 6)

Question from the 2009 FRM practice exam.

14. A Sharpe ratio equals $\frac{R_P - R_F}{\sigma(R_P)} = \frac{9.3\% - 3.2\%}{13.5\%} = 0.452$

While Sortino ratio equals $\frac{R_P - R_F}{\sigma_L(R_P)} = \frac{9.3\% - 3.2\%}{8.4\%} = 0.726$

Tracking error is used to calculate the value of the information ratio, which is defined as:

$$\frac{R_P - R_B}{\sigma(R_P - R_B)}$$

The calculation of information ratio is not required in this question.

$$0.726 - 0.452 = 0.274$$

- B. Incorrect. $2.178 - 0.452 = 1.727$
C. Incorrect. $0.726 - 0.073 = 0.653$
D. Incorrect. $0.73 - 0.452 = 0.378$

(See Topic 6)

Question from the 2009 FRM practice exam.

15. B Once a firm is in distress, it is not in the interests of shareholders to reduce risk. If the firm stays in distress and eventually defaults, shareholders will end up with worthless shares. In these circumstances, management intent on maximizing shareholder value will seek out new risks.
(See Topic 8)

Question from the 2008 FRM practice exam.

16. D 9.08%
This question is really a test as to whether the candidate knows the components of the Sortino ratio.

$$\text{The Sortino Ratio} = \frac{\text{Average Portfolio Return} - \text{Risk-free Rate}}{\text{Semi-standard Deviation (SSD)}}$$

$$0.82 = \frac{12.2 - 4.75}{\text{SSD}}; \text{SSD} = \frac{12.2 - 4.75}{0.82} = 9.08\%$$



Professor's Note: Recall that the Sortino Ratio measures excess return as the difference between the expected portfolio return and a minimum acceptable return. Since you were not given a minimum acceptable return, you had to assume the risk-free rate was the lowest acceptable rate of return.

(See Topic 6)

Question from the 2008 FRM practice exam.

17. A 3.16 bps
- A. Correct. Tracking error is the standard deviation of the difference between the return of the managed portfolio and the benchmark portfolio.

$$\text{TE} = \sigma(R_P - R_B) = \{E[(R_P - R_B)^2] - E(R_P - R_B)^2\}^{1/2}$$
 and

$$E[R_P - R_B] = (4 + (-2) + 6 + 0) / 4 = 2.00$$

$$E[(R_P - R_B)^2] = (16 + 4 + 36 + 0) / 4 = 14.00$$
 So,

$$\text{TE} = (14.00 - 4.00)^{1/2} = 3.16 \text{ bps.}$$
- B. Incorrect. This solution incorrectly sets the tracking error equal to the average difference between the return of the managed portfolio and the benchmark portfolio. Tracking error is the standard deviation of the difference between the return of the managed portfolio and the benchmark portfolio.
- C. Incorrect. This solution incorrectly sets the tracking error equal to the variance of the difference between the return of the managed portfolio and the benchmark portfolio. Tracking error is the standard deviation of the difference between the return of the managed portfolio and the benchmark portfolio.
- D. Incorrect. This solution incorrectly sets the tracking error equal to the difference between the standard deviation of the return of the managed portfolio and the standard deviation of the return of the benchmark portfolio. Tracking error is the standard deviation of the difference between the return of the managed portfolio and the benchmark portfolio.

(See Topic 6)

Question from the 2008 FRM practice exam.

18. C Alpha may be positive or negative depending upon Beta and Risk Free Rate.
- A. Incorrect. Total Portfolio Return equal to Risk Free Return i.e. $RFR + [\text{Beta} \times (\text{Index return} - RFR)] + \text{Alpha}$. This way, Alpha is residual after reducing RFR and Index or Market Related Return from Total Return. It need not be equal to Underperformance Gap of “-2%.”
 - B. Incorrect. If Beta of Portfolio is much lower, Market Related Return will also be lower. This may result in a Positive Alpha in spite of Underperformance.
 - C. Correct. A much lower Beta will reduce Market Related Return and in turn, may increase the residual Alpha to positive figure. Similarly, a higher beta may result in higher share of Market related return implying a Negative Alpha. Hence, Alpha may move anywhere depending upon the levels of Beta and RFR.
 - D. Incorrect. Alpha can be any figure depending upon levels of Beta and RFR. Alpha need not be equal to difference in return of portfolio and index.

(See Topic 6)

Question from the 2008 FRM practice exam.

19. D I and IV
- A. Incorrect. II—The information ratio, not the Sortino ratio, compares the portfolio return to the return of a benchmark portfolio. III—The Sortino ratio allows one to evaluate portfolios obtained through an optimization algorithm that uses semi-variance, not variance, as a risk metric.
 - B. Incorrect. III—The Sortino ratio allows one to evaluate portfolios obtained through an optimization algorithm that uses semi-variance, not variance, as a risk metric.
 - C. Incorrect. III—The Sortino ratio allows one to evaluate portfolios obtained through an optimization algorithm that uses semi-variance, not variance, as a risk metric.
 - D. Correct. I—Since the Sortino ratio uses the notion of semi-variance, it is more appropriate for asymmetric return distributions than any metric that uses standard deviation (such as the Sharpe ratio). IV—The Sortino ratio is similar to the Sharpe ratio, except the risk free rate is replaced with the minimum acceptable return in the numerator and the standard deviation of the returns is replaced with the standard deviation of the returns below the minimum acceptable return in the denominator. II—The information ratio, not the Sortino ratio, compares the portfolio return to the return of a benchmark portfolio. III—The Sortino ratio allows one to evaluate portfolios obtained through an optimization algorithm that uses semi-variance, not variance, as a risk metric.

(See Topic 6)

Question from the 2008 FRM practice exam.

20. C 0.138

$$\text{information ratio} = \frac{\text{average return on the portfolio} - \text{average return on the benchmark}}{\text{tracking error volatility}}$$

$$IR = \frac{13.2 - 12.3}{6.5} = 0.138$$

This question tests whether the candidate knows the information ratio. It has a number of distractors that make it difficult to “guess” the formula.

(See Topic 6)

Question from the 2008 FRM practice exam.

21. B In order to maximize the value, a firm must hedge its financial exposure irrespective of its capital structure.
- Hedging is not mandatory (i.e., hedging could help some firms to increase shareholder value, while for other firms, leaving exposures unhedged or selectively hedged while maintaining more equity may be the value-maximizing strategy). Therefore, consideration of capital structure plays a vital role in hedging decisions.
- (See Topic 8)

Question from the 2008 FRM practice exam.

22. A Long-Term Capital Management
- A. LTCM was an example of strategies that were deliberately undertaken and approved but that didn't pay off. LTCM was subject to operational risks like model risk, but the trades that led to the losses were not unauthorized.
- B. Allied Irish Bank involved a rogue trader making FX trades.
- C. Sumitomo's rogue trading in copper killed it.
- D. Daiwa had a fixed income rogue doing unauthorized trades.
- (See Topic 9)

BOOK 1 FORMULAS

FOUNDATIONS OF RISK MANAGEMENT

expected portfolio return: $\sum_{i=1}^N W_i E(R_i) = E(R_P)$

standard deviation of a two-asset portfolio: $\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \text{Corr}_{AB} \sigma_A \sigma_B$

$$\text{Beta}_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

capital asset pricing model: $E(R_i) = R_F + \text{Beta}_i [E(R_M) - R_F]$

capital market line: $E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_P$

CAPM with personal taxes: $E(R_P) = R_F + \beta_i [(E(R_M) - R_F) - \tau(\delta_M - R_F)] + \tau(\delta_i - R_F)$

where:

δ_M = dividend yield of market portfolio

δ_i = dividend yield for stock i

τ = tax factor that measures market tax rates on both capital gains and income

multi-beta CAPM: $E(R_i) - R_F = \beta_{iM} [E(R_M) - R_F] + \beta_{iI1} [E(R_{I1}) - R_F] + \beta_{iI2} [E(R_{I2}) - R_F] + \dots$

Treynor measure: $\left[\frac{E(R_P) - R_F}{\beta_P} \right]$

Sharpe measure: $\left[\frac{E(R_P) - R_F}{\sigma_P} \right]$

Jensen's alpha: $\alpha_P = E(R_P) - R_F - [E(R_M) - R_F] \beta_P$

Sortino ratio: $\frac{E(R_P) - R_{\min}}{\sqrt{\text{MSD}_{\min}}}$

APT model: $R_n = R_F + X_{n,1} \times b_1 + \dots + X_{n,k} \times b_k + u_n$

where:

R_n = returns for stock n

R_F = risk-free rate

$X_{n,k}$ = k factor exposure for stock n

b_k = return for factor k

u_n = idiosyncratic return for stock n

APT expected excess return: $E(R_n) = R_F + X_{n,1} \times m_1 + \dots + X_{n,k} \times m_k$

where:

m_k = forecasted factor return for factor k

USING THE CUMULATIVE Z-TABLE

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of \$5.00 and a standard deviation of \$1.50. What is the approximate probability of an observed EPS value falling between \$3.00 and \$7.25?

If $\text{EPS} = x = \$7.25$, then $z = (x - \mu) / \sigma = (\$7.25 - \$5.00) / \$1.50 = +1.50$

If $\text{EPS} = x = \$3.00$, then $z = (x - \mu) / \sigma = (\$3.00 - \$5.00) / \$1.50 = -1.33$

For z-value of 1.50: Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z-value of -1.33: Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of -1.33 is $1 - 0.9082 = 0.0918$.

The area between these critical values is $0.9332 - 0.0918 = 0.8414$, or 84.14%.

Hypothesis Testing – One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0%. Assume the population standard deviation is 20.0%. Can we say with 95% confidence that the mean return is greater than 0%?

$H_0: \mu \leq 0.0\%$, $H_A: \mu > 0.0\%$. The test statistic = z-statistic = $\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
 $= (2.0 - 0.0) / (20.0 / 6) = 0.60$.

The significance level = $1.0 - 0.95 = 0.05$, or 5%.

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative z-table. The closest value is 0.9505, with a corresponding critical z-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0 .

Hypothesis Testing – Two-Tailed Test Example

Using the same assumptions as before, suppose that the analyst now wants to determine if he can say with 99% confidence that the stock's return is not equal to 0.0%.

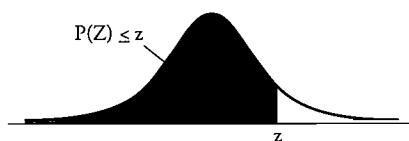
$H_0: \mu = 0.0\%$, $H_A: \mu \neq 0.0\%$. The test statistic (z-value) = $(2.0 - 0.0) / (20.0 / 6) = 0.60$.
The significance level = $1.0 - 0.99 = 0.01$, or 1%.

Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 ($1.0 - 0.005$) in the table. The closest value is 0.9951, which corresponds to a critical z-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0%.

CUMULATIVE Z-TABLE

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$

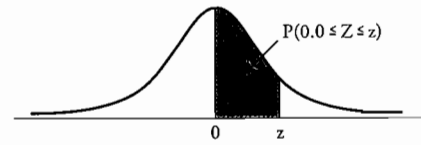


z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

ALTERNATIVE Z-TABLE

$P(Z \leq z) = N(z)$ for $z \geq 0$

$P(Z \leq -z) = 1 - N(z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3356	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4939	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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Notes

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